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# Chaos Theory and the Fall of the Aztec Empire

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**Abstract**

Mathematics has given science a direct way to express some of their laws and relations, but it has also exerted a deep influence indirectly, as a source of associations, analogies and metaphors, i.e., it offers some ways to indicate, intuitively things that can not, or should not, be said. After some general points regarding this problem, a small part of an epic written by the author about the end of the Aztec empire is analyzed as a metaphor where a branch of mathematics, chaos theory, plays a fundamental role.

## Introduction

Mathematics has been a paradigm for scientific thought even since Greek times. Euclid's Elements were so influential that even some philosophical works, like Spinoza's Ethics, were conceived in "ordine geometrico". The axiomatic method has been thought as the ideal way to present a physical theory, and there are many other examples.

It is certainly true that mathematics has seemed to be the only academic discipline where general "true" affirmations could be established. A physical or chemical experiment could be made 100 and 1000 times with the same result, and a very high plausibility is established about the end result of that experiment, but it is based on a finite number of specific cases, and there remains always a small or big doubt about its generalization.

But, on the other hand, I do not measure any real triangle to affirm that the sum of its internal angles is $\pi$ radians, I deduce it, and no matter which angles I measure, if they are the internal angles of a triangle, their sum will be precisely that.

We know now that even if mathematics continues to have some special status, it is not the platonic world of "absolute truths", and it is, in its way, as relative as any other discipline; for after all, it is as human as any other one.

It is well known that mathematics functions as a general paradigm, but there are some other related aspects that have not received the same attention. Mathematics has been the source of several metaphorical ways in what we could call the "non - mathematical world".

We have first mathematics as a reference for two aspects:
a) Exactness,
b) Abstraction, in a negative sense, as something in conflict with practical daily life.

Both are based on a misconception of mathematics. They are not our main concern, but maybe we should explain what we mean.

"This is mathematically exact!", is a common expression, in this or in some related form. But exactness is not a mathematical concept! Two plus two, to use the trivial example, can not be "exactly" four, because it is four, not less, not more. A measurement can have a degree of exactness because it can vary, according to the measuring instrument or to some physical circumstances. Thus, we can be "exact" or not when we measure or construct, but we can not be exact when we add two plus two, because there is only one answer, and no other possibilities.

In the other metaphoric sense, we have also heard countless times someone saying something such as "I do not understand mathematics, it is too abstract, and I am a practical man", meaning that mathematics (and of course mathematicians) are something far away from the "real" world. This is clearly a misconception of the role of abstraction, a tool to get inside reality, and not to get away from it.

## Metaphors

But not everything about mathematics is wrong in the non-mathematical realm. This discipline has been a fruitful source of wonderful and useful metaphors for many other human activities. First it was in geometry. From "linear thinking" to "vicious circle" it has given us many different kinds of expression about ourselves and the world. It was in the Renaissance with perspective that geometry reached a peak. It was giving us "truth" in painting.

Arithmetic has also given us not only the possibility to count, but also a way to look at the world. To build an average, for example, is something arithmetically defined, but, how many times do we speak of an "average man" or the" average woman", even if it is impossible to add all men or women and divide them!

Many of the popular uses of statistics are only a disguised way to say something that is certainly not in the used numbers. We know very well that many applications would not pass a scientific examination.

## Models

Almost everybody is doing today some modeling, or at least, we all say so. But, do we really?

Reality is complex, and extremely interconnected. Our models are not so complex, and are certainly isolated. Well, that is not the problem. The difficulty does not lie in the mathematical systems, but in the use they are given. We believe that our model reflects reality, when, in fact, many models are only analogies, and this is the technical word to speak about metaphors.

Due to the fact that most of the models used are computer models, numerical models, and as such only approximations, this situation only get worse. It is only a matter of reading most of the papers where mathematical models are used in practical situations to discover that almost no one has the smallest concern about the convergence of his or her approximate, numerical solution to the solution of the original system.

We are not trying to say that all these people are wrong, we are only pointing to the power of mathematics as a wonderful generator of metaphors (or analogies).

## Chaos theory

One of the most fruitful mathematical metaphors in the present days is chaos theory. It is fulfilling the same role that catastrophe theory had some years ago. It is used in physics, economics, sociology, remote sensing, etc.

For many of those employing it, it is almost a surprise to know, for example, that chaos theory is strictly deterministic. They are even more surprised when they hear that the mathematical definition of chaos has little in common with the corresponding intuitive notion.

Again, there is nothing fundamentally wrong here, but, as is the case with the use of models, it is much better to be conscious of the real meaning, of the real relationship between that which we try to say and have no precise words for, and the metaphor being used to bring it out, through our feelings, intuition and experience.

It is in this sense that we will try to explain how a mathematical metaphor was employed in a literary work of the author.

## Mathematics and poetry, an example

Almost five hundred years ago, the big Tenochtitlan, the proud capital of the Aztec empire, was conquered and destroyed by Spanish soldiers commanded by Hernan Cortes. This story has been told many times, first by soldiers, then by historians, but it had never been sung, although it is as epic and tragic as the fall of Troy. I tried to do so, and as a mathematician, my thoughts were guided on some occasions by my knowledge and training. My only advantage was precisely this knowledge.

At the end, when the city falls, everything seems to be over, destroyed, but there is some underground river of culture and traditions that can not be annihilated. Very soon a basic thought of chaos theory came to my mind: where apparently no order can be found, there is a second level order. How could that be expressed? How could be said that, under Tenochtitlan's destruction many things survive, or, even better, something new was there?

Here is my answer, in a rough translation [1, pp. 65, 66]:

"From its deepest flight the eagle freezes the image of the battle with its sight. Slowly, a dark nucleus of sliding blood clots outlines itself and, moving, generates a chaotical symbol that turns again and again, with obstinacy, to three places in the center of the city. In each of these places, singled out by the mad directions of knives, swords, obsidian, hardened woods, flesh-impregnated spears, a point stands out, only visible from the fire drunken clouds. In each of these points there is a hairless dog, burning with destiny.

A fourth dog, a gray one, runs from one place to another, between the legs of the fighters, the agile extremities covered with blood and mud up to its buttocks. It runs endlessly the whole day. When the sun nears the extreme bank of the lake the dog finds the most hidden dead bodies. With its whole

strength it transports four bodies, one after another, to the waterway flowing in front of the great pyramid. One of our warriors, one of our women, one of the enemies and one of the allies of the enemy, those from the city of the flowering war. Its jaws tremble already from weariness, the skin of the bodies shows the wounds caused by its teeth. The big esplanade is empty, the battle has gone east. The four bodies are thrown in the water. In the South, an island of flowers has broken its ties and comes on time to go with the dead in their journey".

The epic goes on, but here we can recognize some of the characteristics of chaos theory. We have three attractors, the three points where the dogs stay. We have an indication of the vector field around the attractors, the points of the spears, swords, and obsidian knives. Everything is dynamic, as the fourth dog indicates, and everybody gives his and her own blood, the four bodies, belonging to all parties involved in the battle. I was trying to indicate a deep order, an order surviving the battle and even created by the battle. I did not want to be very explicit, even if I used the word "chaotic", a "chaotic symbol", to gave a hint about what was going on in the description. Mathematics, in its deep relation to so many things and situations, can be, and is, a powerful source of poetry.

## References

[1] J. Carrera Bolaños, Sobre la muerte de la ciudad del valle germinará mi canto, Bibliofilio Mexicana, México, 1997.