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The Marriage of Mathematics and Psychology: An Enduring Love Story

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Abstract

Beneath the popular conception of psychology is a foundation that is composed of mathematics. Unfortunately, few readily see the connection between the two disciplines. In fact, most students of psychology are astounded to discover that they must learn mathematics in order to understand psychology. Mathematics is used to express various psychological theories so that the information can be systematically organized and tested. Mathematics permeates several sub-disciplines within psychology, such as learning, intelligence, decision-making, and neuronal development. It is within this context that the future of psychology lays. This paper will explore the utility of mathematics to psychology, several applications of mathematics within psychology, and how mathematics and psychology, along with computer science, will become even more interdependent in the future.

Introduction

"This is a psychology class, not a math class!" my students cry as I put the Rescorla-Wagner model of associative learning ($\Delta V_n = \alpha\beta\,(\Lambda - V_{n-1})$) on the chalkboard. Soon they learn that to separate mathematics from psychology is to create a false dichotomy. All behaviors are guided by general rules and although we are still trying to discover what many of those rules are, we are finding that mathematics is an important tool to aid in the process of stating the rules of behavior. My goal in this paper is to demonstrate the necessary union between these two seemingly disparate disciplines: mathematics and psychology. I will demonstrate the utility of mathematics to psychology, explore how psychologists have used mathematics, and explain why it is important to maintain the relationship between mathematics and psychology.

What can mathematics contribute to psychology?

> "The impact of mathematical models on the psychology of learning reaches beyond mathematical description, it formalizes variables and their functional relationships" [7].

Psychology is the science of behavior and mental processes. As with all sciences, theories are developed to organize research findings and to aid in making predictions about behavior and mental processes. This organization of research findings and the ability to make accurate predictions about behavior is a daunting process given the complexity of the human organism. However, many psychologists have discovered that by using mathematics to state the relationships in their theories, they can achieve a higher lever of clarity and precision in their theories [3]. The clarity and precision come from a need to explicitly state and define each variable before it can become part of the model.

Mathematical models also provide versatility and generalizability of theories [3, 9]. As we obtain more information about what contributes to various psychological phenomena, mathematical models afford us the opportunity to easily incorporate the new information into the already stated relationships rather than having to rewrite the theory. And, to the extent that similar relationships exist for different phenomena, the abstract character of mathematics allows us to explain the different phenomena with one theory. One poignant example of this generalizability occurred with the Rescorla-Wagner model that was stated at the beginning of the paper. The Rescorla-Wagnermodel was developed to predict an organism's behavior during trials of associative learning (a.k.a. classical or Pavlovian conditioning). This model accurately depicts the negatively accelerated learning curve that typifies the organism's behavior during a series of associative learning trials. A similar relationship was found for the changes that occur in a neuron as an organism is learning – the formation of new connections between neurons, to a certain extent, also occurs in a negatively accelerated manner during the learning trials. Consequently, a mathematical model that was originally conceived as a model to account for changes in *behavior* has been applied to the formation of *neural networks*.

In addition to the clarity, precision, versatility, and generalizability of mathematical models, these models of behavior can elegantly depict complex relationships and they serve as a catalyst for innovative research. T. W. Wickens asserts that it is difficult to rigorously test a theory that is stated in words only because it is nearly impossible to organize the information and visualize the relationships. Mesterton-Gibbons indicates that "Mathematics enables you to deduce... conclusions which (a) might otherwise not be so readily apparent and (b) can be compared with observations of the real phenomenon" [9]. That is, when there are many variables that contribute to a particular phenomenon, such as self-esteem, a mathematical model will aid in organizing those variables so that psychologists can then make predictions about the phenomenon and test whether or not their predictions are correct.

How have psychologists used mathematics?

The formal study of psychology is still relatively young (psychology's birth date is commonly given as 1879, the establishment of Wilhelm Wundt's laboratory) though even while psychology was still in its infancy, psychologists understood the importance of mathematics to psychology. Clark L. Hull was one of the more popular early psychologists to develop a mathematical model of behavior. Hull was interested in what contributes to the motivation of an organism to perform given behaviors; from this, he developed his Drive Theory (${}_{S}E_{R} = {}_{S}H_{R} \times D \times K - I_{R}$) which quantifies the variables that are responsible for influencing the strength of a response to a given stimulus. Several other behavioral theorists have followed in Hull's footsteps; for example, R. J. Herrnstein developed the Matching Law ($R_A / (R_A + R_B) = r_A / (r_A + r_B)$). The matching law expresses the relationship between how an organism will allocate its time given the contingencies established for various activities. William K. Estes developed the stimulus sampling theory ($p_{n+1} = (1 - \theta)\, p_n + \theta$). Stimulus sampling theory is a stochastic model of behavior which indicates that over the course of learning trials an organism begins to substitute adaptive responses for non-adaptive responses and thus learns to perform the responses that will result in the greatest amount of reinforcement.

The integration of mathematics with psychology is not limited to behavioral psychologists, psychometric testing is solidly grounded in mathematics and was part of the early efforts to implement the use of mathematics in psychology. Theories of intelligence and the ways in which intelligence is measured are heavily laden with mathematical models [13] beginning with the Alfred Binet's work on measuring intelligence and creating the concept of a "mental age". This concept was co-opted by William Stern into what we currently know as one's intelligence quotient or IQ (IQ = (mental age/ chronological age) X 100). But perhaps more important to our understanding of the complexity of intelligence was the work of Charles Spearman who implemented the use of factor analysis to arrive at a theory of general intelligence or "g". All popular theories of intelligence stem from Spearman's use of mathematics to arrive at an intangible psychological construct.

Decision-making and choice-behaviors is another area of psychology in which mathematics proliferates. Anytime an organism has to choose between two or more outcomes, mathematics are an easy way to organize the possible choices. For example, approach-avoidance conflicts have been modeled by P. Koene and J. M. H. Vossen. And, L. H. Berger has developed a mathematical model of choice behavior that places reinforcement contingencies on a continuum and allows the organism to determine which is more important, minimizing effort per reinforcement or minimizing time per reinforcement. J. Lazarus discusses the mathematics of choice behaviors involved in long-term mating prospects. There has been a plethora of mathematical models that account for the behaviors of foragers while they are searching for food [2, 4, 5, 11]. Optimality is the general concept upon which many decision-making and choice-behavior models are built [9]. One assumption of these models is that those organisms whose behavior approaches optimality (keeping costs low and benefits high) will be the organisms who survive and pass on the characteristics of optimality to their offspring. The ability to quantify what an optimal choice in a given situation should be makes mathematics the perfect mate for theories of decision-making.

New mathematical models and new uses of mathematics continue to appear in the psychological literature. For example, Tirsch, Keidel, and Sommer claim that, "numerous mathematical models have been developed in the last years for the interpretation of the self-organized rhythmical behavior of biological and social systems" [12]. Tirsch et al. were modeling the nonlinear environment of the human central nervous system, the complexity of which is almost impossible to organize without the use of mathematical models. Mathematics has also been used to model the complex system of human development [10]. As we become experts at uncovering the functioning and development of the human nervous system and its relationship to genetics and environmental experiences, we also become profoundly dependent on mathematics to organize this explosion of information; this is where psychology is heading.

What does the future hold for mathematics and psychology?

Not only is it impossible to separate mathematics from psychology, it would be counterproductive to do so. A new era of computational psychology is on the horizon, and in this era, psychologists will make use of the tools that mathematics and its related field, computer science, has to offer. The complex models of behavior, intelligence, decision-making, and central nervous system functioning that are being developed require the systematic method of organization that mathematics provides. Conjoined with this is that in order to test the predictions of many of these models, the use of computers is necessary since computer can adequately store and use the lager data sets that are currently being compiled. Therefore, to adequately prepare future psychologists for working in their discipline, we must further expose the hidden marriage between mathematics and psychology.

References

[1] L. H. Berger, *The Interactive Schedule: A Common Conceptualizationfor Ratio and Interval Schedules*. Psychological Record, Vol. 38, pp. 77 – 109. 1986.

[2] C. W. Clark, *Modeling Behavioral Adaptations*. Behavioral and Brain Sciences, Vol. 14, pp. 85 – 117. 1991.

[3] M. A. Croon & F. J. R. van de Vijver. *Viability of Mathematical Models in the Social and Behavioral Sciences*. Berwyn, PA: Swets & Zeitlinger B.V. Publishers. 1994.

[4] E. Fantino & N. Abarca, *Choice, Optimal Foraging, and the Delay-Reduction Hypothesis*. Behavioral and Brain Science, Vol. 8, pp. 315 – 330. 1985.

[5] G. F. Grether, R. A. Palombit, & P. S. Rodman, *Gibbon Foraging Decision and the Marginal Value Model.* International Journal of Primatology, Vol. 13(1), pp. 1 – 17. 1992.

[6] P. Koene & J. M. H. Vossen, *A Catastrophe Model of Approach-Avoidance Conflict.* In M. A. Croon & F. J. R. van de Vijver (Eds.) Viability of mathematical models in the social and behavioral sciences (pp. 31 - 54). Berwyn, PA: Swets & Zeitlinger B.V. Publishers. 1994.

[7] P. F. M. Kop & L. F. W. de Klerk, *Influential Mathematical Models in the Psychology of Learning.* In M. A. Croon & F. J. R. van de Vijver (Eds.) Viability of mathematical models in the social and behavioral sciences (pp. 143 – 162). Berwyn, PA: Swets & Zeitlinger B.V. Publishers. 1994.

[8] J. Lazarus, *The Logic of Mate Desertion.* Animal Behavior, Vol. 39, pp. 672 – 684. 1990.

[9] M. Mesterton-Gibbons, *A Concrete Approach to Mathematical Modeling.* New York, NY: John Wiley & Sons, Inc.. 1995.

[10] P. C. M. Molenaar, *On the Viability of Nonlinear Dynamics in Psychology.* In M. A. Croon & F. J. R. van de Vijver (Eds.) Viability of mathematical models in the social and behavioral sciences (pp. 13 – 30). Berwyn, PA: Swets & Zeitlinger B.V. Publishers. 1994.

[11] D. W. Stephens & E. L. Charnov, *Optimal Foraging: Some Simple Stochastic Models.* Behavioral Ecology and Sociobiology, Vol. 10, pp. 251 – 263. 1982.

[12] W. S. Tirsch, M. Keidel, & G. Sommer, *Time Order in Brain Chaos.* In M. A. Croon & F. J. R. van de Vijver (Eds.) Viability of mathematical models in the social and behavioral sciences (pp. 55 – 78). Berwyn, PA: Swets & Zeitlinger B.V. Publishers. 1994.

[13] F. J. R. van de Vijver, *Factor Analysis and Intelligence.* In M. A. Croon & F. J. R. van de Vijver (Eds.) Viability of mathematical models in the social and behavioral sciences (pp. 163 – 184). Berwyn, PA: Swets & Zeitlinger B.V. Publishers. 1994.

[14] T. W. Wickens, *Models for Behavior, Stochastic Processes in Psychology.* San Francisco, CA: Freeman. 1982.