BRIDGES
Mathematical Connections
in Art, Music, and Science

Come un meccanismo di precisione: The Third Movement of Ligeti's Second String Quartet

Diane Luchese
Department of Music
Towson University
Towson, MD 21252-0001, U.S.A.
E-mail: dluchese@towson.edu

Abstract

The third of the five movements that comprise György Ligeti's (1968) Second String Quartet bears the performance instruction "come un meccanismo di precisione," or "like a precision mechanism." More than just a performance instruction, these words describe the image that the music conjures, as well as the composition itself. This paper is an analytical presentation of the movement which highlights Ligeti's use of golden section proportions, symmetry, numerical series, and permutation.

Introduction

György Ligeti's fascination since childhood with recalcitrant machinery comes to creative fruition in his musical compositions in the "pattern-meccanico" style [1]. His pattern-meccanico pieces feature rapidly repeating pitches played without accent, or in a mechanical fashion [2]. A wonderful example of this style of composition is the third movement of Ligeti's (1968) Second String Quartet, which bears the performance instruction "come un meccanismo di precisione," or "like a precision mechanism." More than just a performance instruction, these words describe the image that the music conjures. Functioning formally within this quartet as a scherzo, this movement compares with many traditional scherzos in its light and playful character, its quick tempo, and its three-section form. The movement breaks from tradition by substituting a complex of rhythmic transformations, which result from four individual strands akin to parts of a machine, for idiomatic melodic and harmonic gestures usually heard in string quartet literature. This paper is an analytical presentation of this movement which highlights Ligeti's musical application of golden section proportions, symmetry, numerical series, and permutation.

Formal Divisions and Their Durational Correspondence to Golden Section Proportions

The third movement of Ligeti's Second String Quartet can loosely be described as a ternary form, a musical form that divides into three sections of which two similar outer sections frame a contrasting middle section. The two outer sections of this movement are characterized by pizzicato (plucked) repeated pitches that are to be played evenly and without accent, or, in a mechanical fashion. The middle section marks a contrast, beginning with a bowed sweeping flourish that leads to extremely rapid changes of articulation, pitch, rhythm, dynamics, and tempo. This form can be described using the letters A - B - A^1; the letter "A" that follows "B" indicates a return of the initial musical material, while the "1" (prime) indicates that the reprise is somewhat varied. The sections flow into

one another without cadence or caesura. Thus, the sectional divisions have little to do with rests or breaks in the music and instead result from a contrast of musical material.

Although the three sections are not symmetrically balanced in length, the durations of the sections seem to be modeled on golden section proportions [3]. The golden section is a ratio between two segments of a line, in which the smaller of the two segments is to the larger as the larger is to the sum of both segments, approximately .618 to 1. In this paper, .618 from the beginning of a total length will be referred to as the positive golden section, and .618 from the end (or .382 from the beginning) of a total length will be referred to as the negative golden section (see Figure 1).

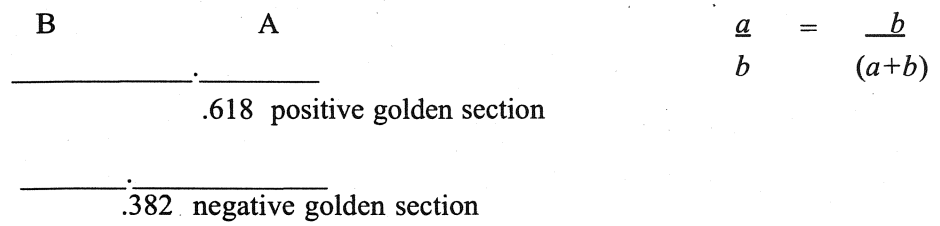


Figure 1: *golden section ratio*

In this discussion, the concept of "line" is being applied to the composition's "length" in clock time. By examining the lengths of the movement's three sections, one can see that the formal divisions approximately correspond to golden section proportions (see Figure 2). The positive golden section of the total length falls within five seconds of the seam between the A and B sections. If we separately examine the combined length of the B and A1 sections, we see that A1 begins at the negative golden section of this time segment.

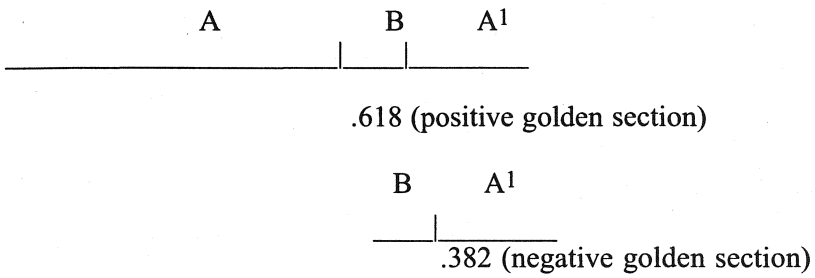


Figure 2: *formal divisions corresponding to golden section proportions*

Examining each individual section likewise reveals golden section proportions modeled by subsectional divisions. The A section subdivides into two parts, marked by a shocking snap pizzicato in the cello and a short rest interrupting the activity, which are followed by a shift to a lower range of pitches. This moment occurs at the negative golden section (Figure 3a). The B section subdivides into two parts, also marked by a snap pizzicato in the cello. The bowing in the first subsection is contrasted by what sounds like a series of pops and snaps in the second subsection. This subdivision occurs at the positive golden section (Figure 3b). The final A1 section resists subdivision, sounding as one continuous arch. Nevertheless, .382 of the way through this section is an unusual convergence of all four strings on a single pitch. This is the final pitch event in the piece and is prolonged for the remaining .618 of the section's length (Figure 3c).

(3a) section A _____._____
.382 (snap pizzicato marking subsectional division)

(3b) section B _____._____
.618 (snap pizzicato marking subsectional division)

(3c) section A^1 _____._____
.382 (instruments converge and remain on single pitch)

Figure 3: *subsections corresponding to golden section proportions*

Section A

After two initial measures of silence, the mechanism begins its processes in the form of quickly plucked repeating pitches. Listeners attend to changes of the speed of the plucking (tempo), changes of the specific pitches being plucked, changes of register (the relative highness or lowness of the range of the plucked pitches), and changes in the width of registral space (the encompassed width of the range of pitches). As mentioned above, an extremely loud snap pizzicato subdivides the section into two parts at the negative golden section. The two subsections exhibit similarities and contrasts.

Contour, Register, and Registral Space: Symmetrical Properties [4]. The profile of the first subsection of Section A is an ascending contour that enters the next higher octave. The initial registral space is a narrow width that first expands and then partially contracts (see Figure 4). The first violin and the viola begin plucking in unison on the pitch B^5 while the second violin and cello are plucking the pitch a whole step below, A^5 [5]. Not only is the cello playing in an unusually high register, but the interval of a whole step (two semitones) is an unusually narrow registral space for a string quartet, given the instruments' potential to encompass a tremendously wider range of pitches. Gradually the pitch range of the instruments spreads apart, opening the registral space to a width of fourteen semitones, and then narrows in half to a span measuring seven semitones. The cello reaches the upper limit of its range just preceding its snap pizzicato interruption on a low B^{b2}.

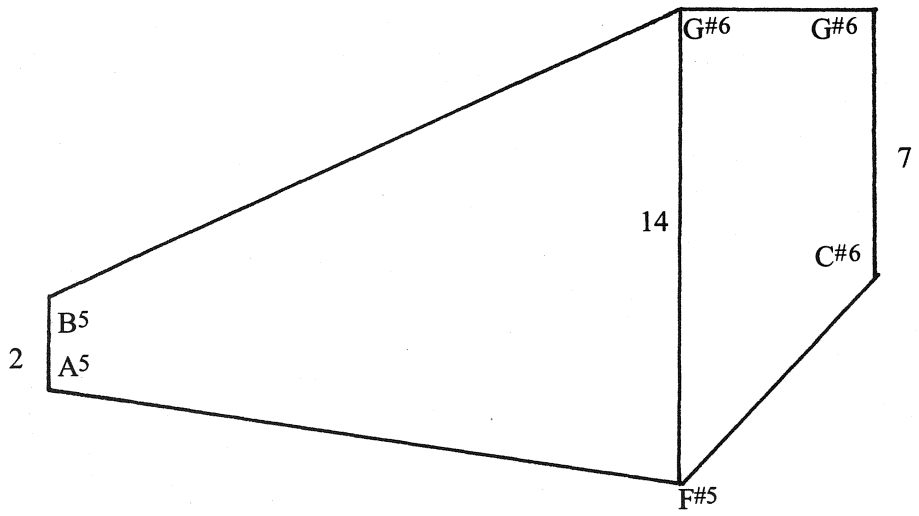


Figure 4: *profile of general contour and registral space of the first subsection*
(numbers refer to the size of the registral space measured in semitones) [6]

The profile of the second subsection contrasts the first subsection with its overall lower register and contour, which essentially descends into the next lower octave. It begins similarly in a narrow registral space that expands and then contracts, although it begins slightly wider, expands even further, and then contracts to an even narrower space than does the first subsection. As this subsection begins, the strings resume plucking on four individual pitches {D4, Eb4, F4, F#4}, a narrow span that measures four semitones. The lines then spread apart and the registral space eventually expands to 30 semitones. This expansion is coupled with increased textural density: each instrument plays two pitches simultaneously (double stops), producing a collective sonority composed of eight different pitches. Next, the textural density thins back to four tones and the upper parts descend. By the end of the section, the violin is near the bottom of its range, and the narrowed space approximates the width of two semitones (see Figure 5). The space falls within the pitch boundaries of↓A3 an↑F#3; the arrows preceding the pitches refer to microtonal pitches that fall slightly below and slightly above the respective specified pitches.

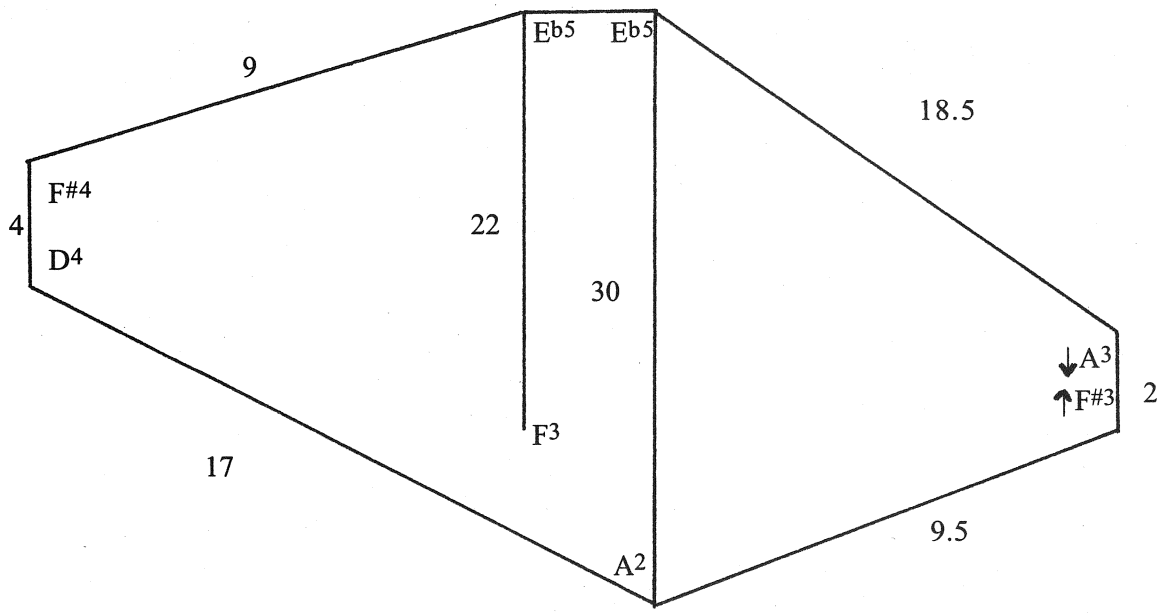


Figure 5: *profile of general contour and registral space of the second subsection*

Preceding the registral expansion to 30 semitones is a preliminary expansion to 22 semitones which unfolds symmetrically. The four instruments can be viewed as two pairs of similarly inverted contours. The second violin ascends a total distance of three semitones while the viola descends a distance of three semitones. The cello descends a span of nine semitones while the first violin ascends a span of nine semitones. The pitches change in ever so slight increments, making use of microtones. Thus, the ascending upper strings and the descending lower ones expand the space in mirror fashion. It should be noted that while the linear motion of these mirrored pairs of lines is concurrent, the changes of pitch are not completely simultaneous. In each pairing, one instrument leads while the other follows. In musical terms, this procedure is described as canonic imitation by inversion, or a mirror canon. These mirrored lines that are slightly out of phase can also be interpreted as "*out-of-phase sonic bilateral symmetry*" (see Figure 6).

Now that we have examined the profiles of the two subsections and seen how they complement one another, a final diagram that combines and summarizes the boundaries of the registral space reveals a fascinating symmetrical profile (see Figure 7). To summarize, the A section began in register five as a span measuring two semitones and concluded in register three as a similarly narrow span. The contour ascended in the first subsection to a high point of G#6, and descended in the second subsection to a low point of A2.

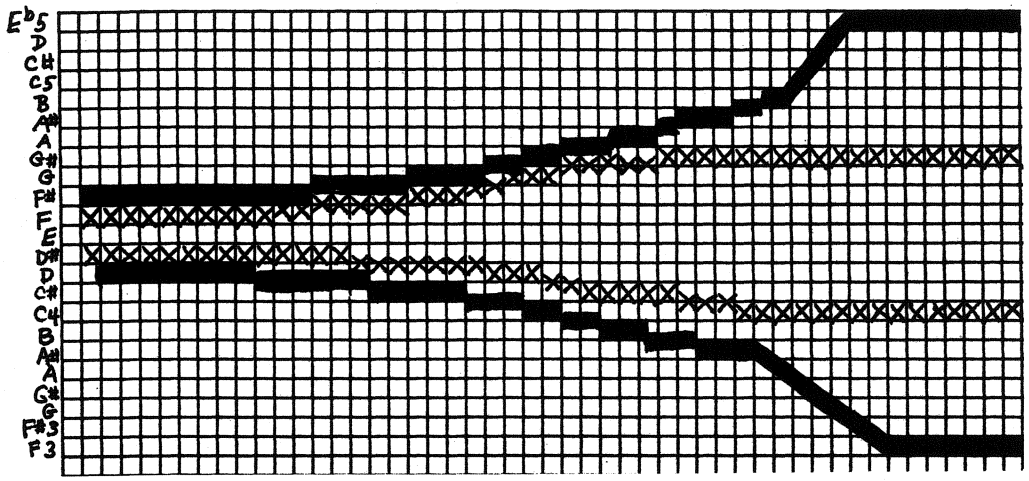


Figure 6: *symmetrical expansion of registral space to 22 semitones*

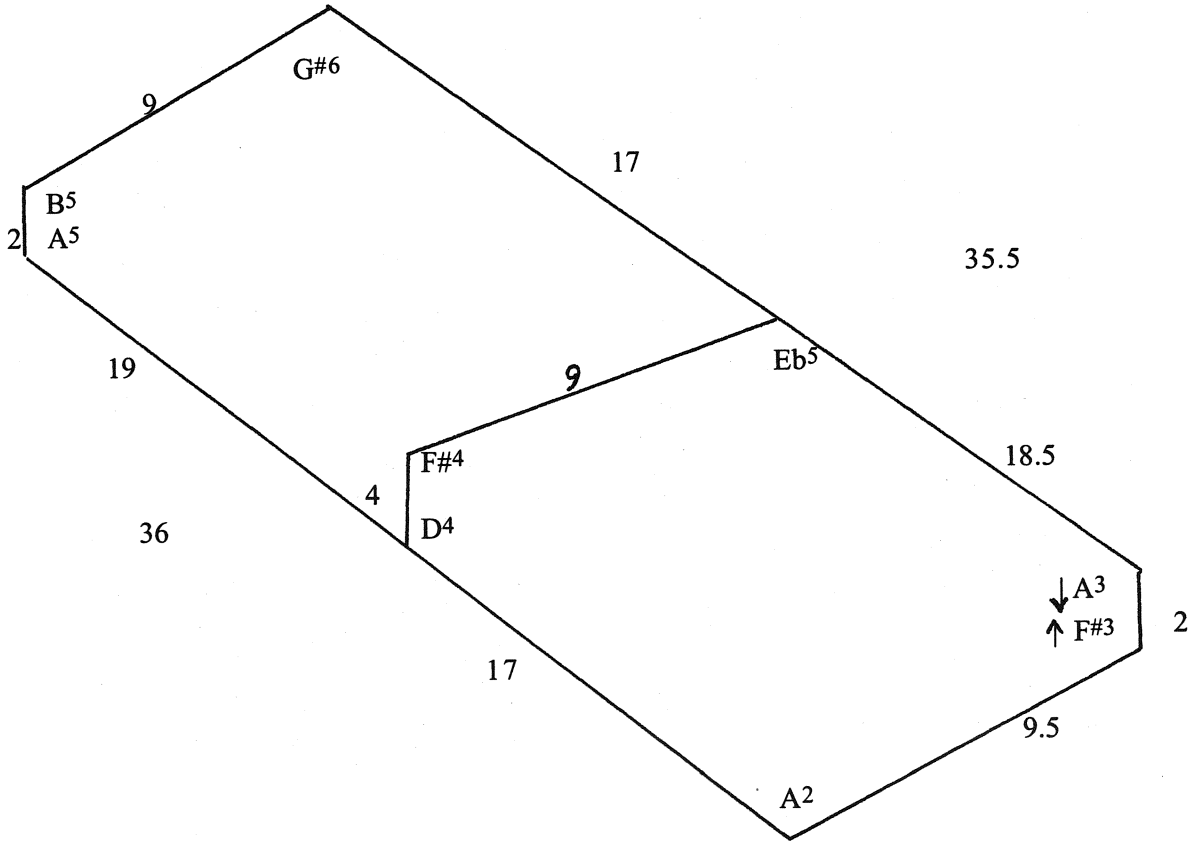


Figure 7: *symmetry among the registral boundaries of Section A*

Changes of Tempo and Phasing: Beat Subdivisions in Numerical Series. While the mechanical plucking of repeating pitches without accent prohibits a perceived sense of meter, salient rhythmic characteristics throughout the A section include tempo changes as well as movement both away from and toward a simultaneous unison rhythm, or, movement out of and in phase. Again, the two subsections are examined individually.

In the first subsection, the four instruments begin in rhythmic unison, or synchronously, and then gradually grow out of phase with one another through individual accelerations. Acceleration is achieved as the rate of plucked attacks increases through a procedure systematically structured on a numerical series. The opening attacks initiate from a musical notation in which a measured temporal unit is subdivided into four. Acceleration begins in the first violin part when the notated subdivision increases from four to five, and continues to twelve in numerical succession. The other voices successively imitate this procedure. This fixed systematic procedure of imitation can be described in musical terms as a four-part rhythmic canon. The effect here is four strands that gradually move out of phase. Once the notes played by the cello reach the speed resulting from twelve subdivisions, the parts begin a very rapid but systematic deceleration, although no longer in imitative fashion. This rapid deceleration is combined with an increase in dynamics and the shift to a higher register, described above. The effect is one of intensification until the dramatic interruption by the cello's snap pizzicato. The following chart (Figure 8) shows the imitative (canonic) acceleration and nonimitative but systematic deceleration of the parts, with the numbers referring to the subdivisions of the temporal unit (the higher the number, the quicker the speed of the plucked pitches).

Vln 1:	4	4	5	6	7	8	9	10	11	12	12	12	12	12	12	11	10	5	5
Vln 2:	4	4	4	5	6	7	8	9	10	11	12	12	12	12	12	6	6	6	6
Viola:	4	4	4	4	5	6	7	8	9	10	11	12	12	12	7	7	7	7	7
Cello:	4	4	4	4	4	5	6	7	8	9	10	11	12	12	11	10	9	8	4

acceleration - - - - - - - deceleration

in phase | - - out of phase - - | in | out of phase

Figure 8: *effects of the unit subdivisions based on number series, first subsection*

After the snap pizzicato, plucking resumes in the second subsection in a lower register. Acceleration again is achieved through a series of successively increasing subdivisions which exceed the rate in the first subsection. The subsection's phasing profile opposes that of the first subsection, and ends with an extended area of simultaneous plucks that subdivide the temporal unit into 16 (see Figure 9). In these last five measures, acceleration continues from the "poco a poco accelerando" and the "as fast as possible" instructions in the score. The notated tempo approximates eight plucked attacks per second. Whatever intensity that might have been gained through this acceleration is countered by the low register, the decreasing dynamics, and the prolongation of unchanging pitches. The effect is instead one of de-intensification, and these final bars seem static.

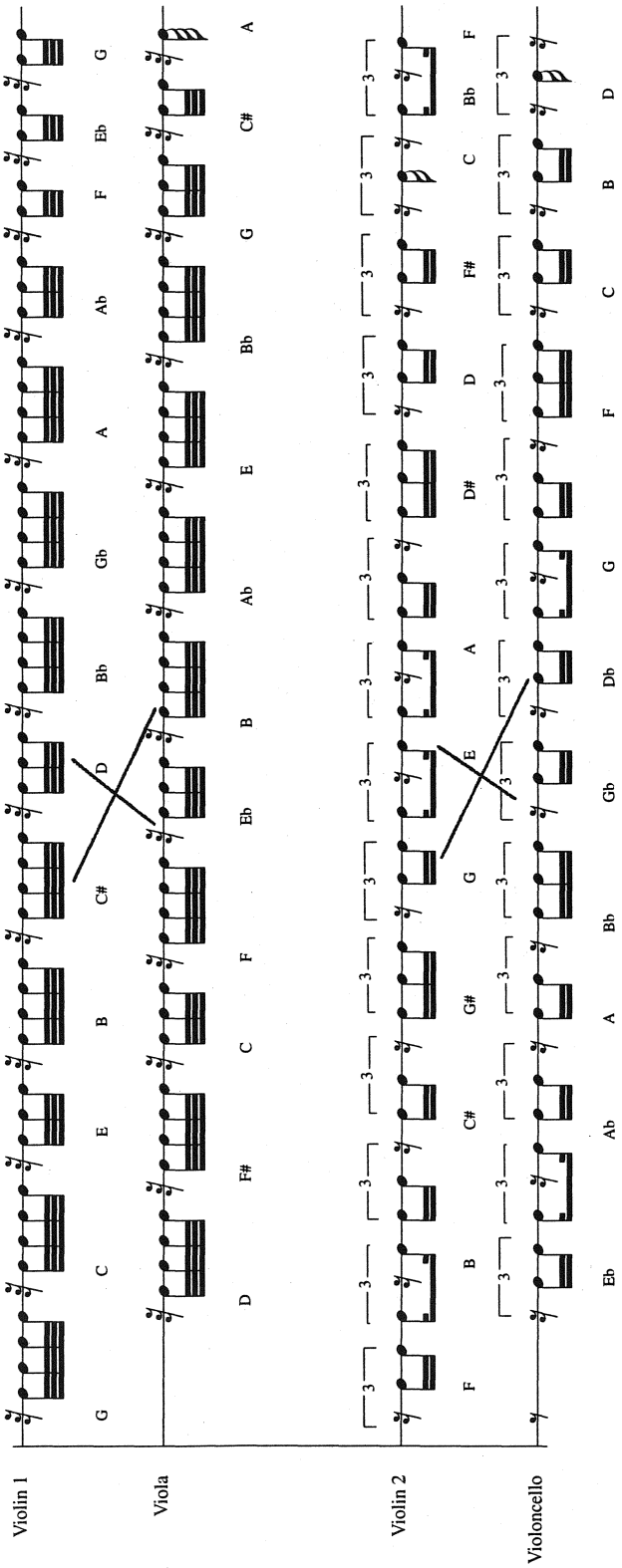
Vln 1: 5 5 6 7 7 7 8 9 10 10 10 10 10 10 10 10 10 11 12 13 14 15 16 16 16 16 16 16 etc
Vln 2: 4 4 5 6 7 7 7 8 9 10 10 10 10 10 10 10 10 10 11 12 13 14 15 16 16 16 16 16 etc
Viola: 7 7 7 7 7 7 7 7 8 9 10 10 10 10 10 10 10 10 10 11 12 13 14 15 16 16 16 16 etc
Cello: 3 3 4 5 6 7 7 7 7 8 9 10 10 10 10 10 10 10 10 10 11 12 13 14 15 16 16 16 etc
acceleration - - - - - - - - -
out of phase - | in phase | - out of phase - | in phase

Figure 9: *effects of the unit subdivisions based on number series, second subsection*

Section B

In Section B, whatever processes engaged by the precision mechanism that were conjured in the listener's mind now seem dramatically different. This short section, lasting approximately 28 seconds, is a compression of musical events exhibiting rapid changes of pitch, register, contour, dynamics, articulation, timbre, and rhythm. Although the shortest of the three sections, it is the most energized and virtuosic, and also contains the largest concentration of events. Like the A

section, it also subdivides into two parts, the first having flowed directly out of what precedes it.



As the B section begins, the prolonged rapid plucking transforms into a flourish in which all the instruments bow a ferociously quick melodic gesture. Within two seconds, the registral space opens from a narrow span approximating a whole step to the widest registral span of the entire movement, a width one semitone less than six octaves (71 semitones). The cello plunges downward to the movement's lowest pitch, C^2, and the first violin ascends to the movement's apex, B^7. One may recall that this section begins roughly at the time corresponding to the golden section.

Following this explosive flourish, the melodic lines are highly disjunct, and consist of nimbly bowed, short groups of staccato pitches which leap to other similarly played pitches. The successive pitches of each string part are composed of different permutations of the set of twelve pitch classes that comprise the chromatic series {C, C#, D, D#, E, F, F#, G, G#, A, A#, B} [7]. Each instrument plays its own entire order of the set of pitch classes before repeating any, as in the practice of serial music (see Figure 10). After the instruments complete their statements, they begin restating their permutations, although not in entirety the second time. Toward the end of the second statements, the bowing switches to loud plucking at a slower speed, and a snap pizzicato played by the cello interrupts the process and marks the subsectional division. It is interesting to note that just preceding this snap, at the positive golden section of Section B, the pitches played {F4, F#4, G4} have a combined registral width of two semitones, the same registral width that framed the A section.

Figure 10: (sideways) *Section B, four permutations of the set of twelve chromatic pitches mapped onto two concurrent imitative rhythms (out-of-phase bilateral symmetry)*

Ligeti maps the four permutations of the set just described onto two concurrent imitative rhythmic procedures. The instruments work in pairs; the first violin couples with the viola and the second violin couples with the cello to play two separate imitated rhythms, or two rhythmic canons. Thus, while the four individual parts state their permutations of the chromatic series, they are paired to play them as short groups of repeated pitches in a loosely structured rhythmic canon. The viola mirrors the rhythmic pattern that the first violin starts, and the cello mirrors the rhythmic pattern that the second violin starts, each following at a short temporal distance. These parallel rhythmic lines are yet another example of *out-of-phase bilateral symmetry* in the rhythmic domain. Because the rhythms of the first pair subdivide the eighth note temporal unit into four, and those of the second pair subdivide the same temporal unit into three, the result is one of polyrhythmic complexity. (One may recall the use of rhythmic canon in Section A for the effect of producing superimposed strands of out-of phase acceleration.) Figure 10 shows a portion of the rhythmic notation of the imitative (canonic pairs), with the concurrent pitch set permutations of each instrument stated below the rhythmic notation. Attention is drawn to the one spot in each imitative pair where the process temporarily goes off course.

The snap pizzicato in the cello, which interrupts the above-mentioned procedures, sets off what sounds like a series of pops and snaps. Have the processes at work gone awry? This curious activity comprises the second part of Section B. The performers, instructed to put down their bows, produce highly contrasting timbres through an array of pizzicato techniques played in a variety of ways including near the bridge and over the fingerboard. The dynamics are varied, and range from very loud and accented to very soft. The pitches played by each instrument collectively constitute one more permutation of the chromatic series. Each instrument seems to take its turn introducing the next pitch class, and together they generate the order shown in Figure 11.

A (cello) - B^{b} (cello) - B (viola) - C (violin 1) - A^{b} (violin 2) - $C^{\#}$ (cello) - D/E^{b} (viola) -

E (violin 2) - $F^{\#}$ (violin 1) - F (cello) - G (violin 2)

Figure 11: *collective permutation of the set of chromatic pitches*

After this pitch series, the viola plays a snap pizzicato (sffff) on A^{b3}, as if to start yet another permutation. But just after two more pitches {B^{b3} and A^{3}} are played within literally half a second, soft mechanical plucking begins the final A^{1} section. The last three pitches of Section B measure the familiar narrow registral width of two semitones, in the same register that originated the flourish at the beginning of the section.

Section A^{1}

Section A^{1} resists segmentation or division, as it is one large arch devoid of phrases, interruptions, or punctuation. This section is completely in a state of decreasing intensity. What begins as a thick texture and somewhat wide registral space narrows to the smallest space possible: convergence on a single pitch. Additionally, the out-of-phase plucks among the strands become synchronous, and then gradually die away in softness.

Throughout the entire section, the cello remains on one pitch, the highest pitch in the texture, $F^{\#5}$. As the cello prolongs this pitch, the other parts ascend to join it. The viola is the first to join the cello on $F^{\#5}$, followed by the second violin, and then the first violin. As stated above, the voices converge onto this single pitch at the negative golden section of the length of this section.

The thinning textural density is easily perceptible. As this section begins, the cello combines with the other three instruments, each playing three pitches (pizzicato arpeggiations), to form a ten-voice texture (3+3+3+1). Soon after, the instruments pluck single pitches, thinning the texture to four voices. As the instruments stop playing successively, the texture further decreases to three voices, then to two, and then to just one voice, the cello. Instruments stop playing in the same order that they join the cello on $F^{\#5}$. Soon after the first violin stops playing, the cello joins the silent parts for the two measures of rest that conclude the movement.

The narrowing registral space is equally perceptible and recalls features of the A section. This section begins at its widest registral span and gradually contracts to a single pitch. The opening span of 30 semitones, C^3 to $F^{\#5}$, recalls the intervallic size of the widest registral span achieved in the initial A section. Then once the texture thins to four pitches, the registral space narrows to 22 semitones. This intervallic size recalls the goal of the symmetrical unfolding that preceded the expansion to 30 semitones in Section A. The registral space continues narrowing to a single pitch. It is interesting to note Ligeti's choice for a final pitch. $F^{\#5}$ was the lowest pitch of the first subsection of Section A. It is also two semitones below $G^{\#5}$, the pitch that corresponds to the golden section of the movement's entire registral space, or total range of pitches [8].

In addition to convergence on a single pitch, the parts decelerate and also converge to rhythmic unison. The strands begin this section non-synchronously as cross rhythms, i.e, 14 unit subdivisions against 12 against 10 against 8. The effect is that of layers playing in *slightly* different speeds. Gradually each instrument arrives at the same speed, just before each stops playing. The progression from out of phase to synchronous plucking recalls the end of Section A, although the general trend here is deceleration instead of acceleration. Figure 12 shows the successive temporal unit subdivisions throughout the section.

violin 1 (decelerates)	14	13	12	11	10	10	10	9	9	8	8	8
violin 2 (accelerates, then decelerates)	8	9	10	9	9	9	9	9	8	8	8	8
viola (decelerates)	10	10	9	8	8	8	8	8	8	8	8	8
cello (decelerates, then accelerates)	12	12	11	10	9	8	7	7	7	8	8	8

Figure 12: *temporal unit subdivisions, deceleration and progression to rhythmic unison*

The softening dynamics, the narrow registral span, and the progression to synchronous playing describe a profile similar to the second subsection of the A section. This time, no rush of activity follows. Instead, the mechanism has shut down.

Concluding Remarks

This examination of the third movement of Ligeti's Second String Quartet revealed formal divisions corresponding to golden section proportions, contour and registral profiles displaying spatial symmetrical properties, melodic lines based on set permutations, procedures of acceleration and deceleration based on number series, and imitative rhythmic procedures displaying temporal symmetry, albeit "out-of-phase." These musical procedures that both significantly shape this movement and direct the listeners' attention also bridge to mathematics. "Like a precision mechanism" is more than a performance instruction; it also describes the image that the music conjures. A mechanism seems to start up, perform some functions with a few interruptions, go awry, and eventually shut down! Having explored the technical processes at work and the composer's brilliant engineering, we can conclude that the words "like a precision mechanism" additionally describe the resultant composition itself.

References

[1] To read about Ligeti's fascination with "unmanageable automata" see Péter Várnai, "Ligeti-Péter Várnai," trans. Gabor J. Schabert in *Ligeti in Conversation with Péter Várnai, Josef Häusler, Claude Samuel, and Himself* (London: Ernst Eulenberg Ltd, 1983), 17.

[2] For a comprehensive discussion of Ligeti's pattern-meccanico compositions, see Jane Piper Clendinning, "The Pattern-Meccanico Compositions of György Ligeti," *Perspectives of New Music* 31, no. 1 (winter 1993): 192-234.

[3] Ligeti has acknowledged his use of golden section proportions. See, for example, Pierre Michel, *György Ligeti: Compositeur d'aujourd' hui* (Paris: Minerve, 1985), 131.

[4] Jonathan Bernard discusses this movement in light of voice leading in a spatial context in "Voice Leading as a Spatial Function in the Music of Ligeti," in *Music Analysis* 13 (1994): 227-253. Bernard also addresses symmetrical features of the first 30 bars of the work.

[5] The octave designations in this paper follow the system recommended by the Acoustical Society of America, whereby middle C, 523.3 cps, is C^4, and tuning A, 440cps, is A^4. C^3 is the octave below middle C, and C^5 refers to the octave above that, etc.

[6] I would like to acknowledge the work of Pozzi Escot, whose numerous and insightful graphic depictions of musical properties have inspired the graphs I include in this paper. For her most recent work, see *The Poetics of Simple Mathematics in Music* (Cambridge, MA: Publication Contact International, 1999).

[7] "Pitch class" refers to a pitch without reference to the specific octave or register of its occurrence. (For example, pitches an octave apart, such as C^3 and C^4, are considered the same pitch class.)

[8] For those troubled by the fact that $F^{\#5}$ does not correspond exactly to the golden section, or by any of the other "approximations" and kinks in his procedures, please note that Ligeti states "I detest both absolute geometrical precision and total openness. I want a certain order, but an order slightly disorganized." Michel, 180.