# Geometric Mosaic Tiling on Hyperbolic Sculptures

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## Abstract

Although I am a non-mathematician my artwork originates with the aesthetic intuition of geometric form. It bears a relation to the natural world and the mathematical coherence found in natural forms. When we look at nature we see patterns. We discover the rhythms of life and see motion in form. Patterns are my metaphor for the structure and hidden formal order of our natural world. As a mosaic artist I make tessellated sculptures and approach the concept of infinity through my art. I construct my universe through surface division and hyperbolic geometry.

The structure and patterns of my hyperbolic sculptures may be analyzed mathematically. Yet, as visual art objects they stand on their own merit and can be appreciated by general audiences.

## 1. Description of Hyperbolic Sculptures

**1.1 Infinity.** The notion of infinity has fascinated me. It suggests timelessness. In my hyperbolic sculptures infinity is metaphorically represented by repeating patterns and regular tessellations, which decrease in size at the bounding edge. In hyperbolic geometry the concept of infinite smallness is contained within the finite circle. There is a reduction in size of the arcs from the larger center to the smaller arc leading from the center to the outer edge. The idea is that of constant diminution. The distance is not measured uniformly over the whole unit disc and therefore the figures are said to be congruent. The interior arcs mark an angle and cross each other in ever-smaller distances. [1] On the hyperbolic plane the arcs of the circles cut the circle and are orthogonal to the bounding edge. They are perpendicular to the unit disc including diameters. Its circumference represents infinity, beyond that is "absolute nothingness". [2] The circles have a constant curvature and are not concentric. The smaller their diameter, the closer their centers are to the bounding rim.disc. They suggest a sequence towards the infinitesimal at the circumference. The curvature of the 3D-mosaic sculpture is hyperbolic (meaning concave) and consists of a tiled pattern surface division. (Figure 1, Figure 2)

**1.2 Time.** The concept of time is represented metaphorically. It is implicit in the subtractive process of cutting away to obtain modules and then piecing these modules together to form a surface pattern. The mosaic surface is composed of units that combine to resemble pixels on a computer screen. It is a part to whole relationship. One becomes aware of the energy and time expended in cutting and piecing together the tiling pattern from point to segment, to shape. The

patterns consist of hand cut mosaic *tesserae*. The individual pieces are cubes and rectangular solids. These *tesserae* are made of marble and glass. The subtractive process of cutting the *tesserae* from the initial large block of stone or glass involves successive steps of cutting and rotating the polygon to achieve the desired module size. It is similar to a jeweler cutting a diamond. These tiny gems of glass and stone are then pieced together one by one (using tweezers) to form a tiling pattern. These patterns repeat in different directions. Consistency is a formal property of the repeating patterns.

The element of time and energy is embedded in the process of making mosaic sculptures. Each mosaic *tesserae* (a module of a cube or rectangular solid the size of a tooth) is a hand cut module which is added to other modules and tightly pieced together to form a surface pattern. The connection to mathematics is that one becomes aware of the process as the time and energy it took to create the artwork. (Figure 3)

**1.3 Shapes: Pentagon and Circle.** The interior of the circle is conformal (preserves the angles) on the hyperbolic plane. Within the interior of the circle is a curvilinear pentagon, which is recessed 3" and is at the center of the sculpture. The angles of the pentagon are doubles of those in the circle. The tessellated surface pattern is a combination of repeating shapes "Hyperbolic Diminution" Sculpture I (Figure 1). In "Hyperbolic Diminution" Sculpture II and III the pentagon has right triangles whose sides are the mirror reflections.

A sequence of alternating shapes that diminish towards the interior central point suggests infinity. The sequence is a circle and a pentagon. The tiled pattern is regular and predictable. Yet in Sculpture I "Hyperbolic Diminution" (Figure 1) the circle within the recessed pentagon has an element of unpredictability and randomness. It is my metaphor for the natural world and a system within which we find both determinism and uncertainty. In Sculpture II and III the pentagons are distorted because it is a metric topological pattern. (Figure 6)

**1.5 Symmetry.** Tiling patterns on the surface of my hyperbolic sculptures have symmetry of design. By that I mean it maps the design onto itself. That is the symmetry is a transformation that preserves the distance. All the points are equidistant and can be mapped onto each other. They are also equivalent in all directions. Symmetries are of four kinds. They are translation, rotation, reflection and glide-reflection. The classification of the design patterns can be achieved using symmetries groups and the number of shapes in the patterns. In Islamic and Moorish art "periodicity is a constant factor in the tessellations". [4] The tiling pattern in Sculpture II is semi regular {4.5.4.5} (Figure 6) and showing regular {5.4 } tessellation in Sculpture III. It is strictly hyperbolic tiling of the plane.

## 2. Construction of Sculpture.

**2.1 3D Wood model.** Douglas Dunham, the distinguished computer scientist, was very helpful and provided me with Pointcaré's model of line drawings and reproductions of his own computer generated prints in the style of M.C. Escher's famous woodcuts the *Circle Limit* series. My challenge was to construct an aesthetic three-dimensional art object using a two dimensional flat line drawing. In Euclidean geometry they are two halves of a single line but they form an angle on a hyperbolic plane. It was an arduous task to realize it as a wall sculpture whose primary function is an aesthetic art object. (Figure 4, Figure 5)

**2.2 Pointcaré Model.** Hyperbolic geometry was invented by Pointcaré. Pointcaré's model is conformal which means it preserves the angles. In the Euclidean size and the hyperbolic size the

spaces decrease in proportion to the distance to the edge and the center grows logarithmically while the distance to the edge becomes the inverse. [5]

In the hyperbolic model the distances can be understood as a person standing in the center of the circle and walking towards the outer edge. The distances are the same size for the walker. But for the observer standing outside the bounding edge the walker's distances from the center are progressively smaller. [6]

**2.2 Geometric Mosaic Tiling: Materials and Technique.** My hyperbolic sculptures have a tessellated surface of marble and glass. The word tessellated in art and a non-scientist's point of view describes a pieced together surface pattern of small similar pieces. I use a *martellina*, which is a small hammer with two cutting blades and a *tagliolo*, which is a hardie. The materials I use are stone and glass. I place the piece of marble or glass on the hardie and strike it with a blow. Each cut requires a rotation in order for it to be a cube or rectangular solid. These little gems of marble and glass are called *tesserae*. They are the size of your tooth or fingernail. They are like pixels on a computer screen, which I pieced together with tweezers one by one to form a surface pattern.

Glass is a man-made substance. It is made by fusing silica-like sand with an alkali such as soda and potash. Colors are obtained by the addition of metallic oxides. For example copper produces green and blue-greens, cobalt results in a vibrant blue, iron makes pinks and yellows, manganese becomes black and browns, and zinc becomes a white.

Marble is crystalline limestone, which can be polished to achieve a high gloss finish. In nature it can be found in a variety of colors with interesting graining.

Traditionally mosaics have been made from glass, stone and glazed tile. Marble has been most evident in early Greek and Roman buildings and mosaic art. While geometric mosaic patterns are found throughout the ancient world, in later civilizations the figurative style became a dominant mode with geometric designs playing a secondary role. However unlike their Byzantine precedent the Islamic designers began to use mosaic in a non-figurative manner. They used marble carvings with intricate lace like patterns. The Alhambra attests to the skills of the Islamic masons and sculptors.

The use of geometric brick mosaics is found throughout the Islamic world. Mosaic like patterns using sun-dried clay bricks as well as glazed bricks have been used as a building material to cover the exterior of the building. The use of glazed and fired clay bricks is more durable as a structural material. [7] They are laid in a mosaic like patterns and are often geometric and decorative. Some include religious quotations from the Koran.

In Moorish architecture geometry is a "direct expression of the divine unity" [8] (Fractions are not used). Although numbers were used as a means of measurement and calculations in the ancient civilizations such as the Greeks, and Euclid, it developed into an important component in Islamic mathematics. Mosaic materials and technique have a rich history in architecture and art. The mathematical components in Moorish patterns from the Alhambra inspired M.C. Escher and continue to be of interest to computer scientists exploring geometric tiling patterns and designs.

## 3. Conclusion

Geometric patterns are my metaphor for the rhythms and structure I find in the natural world. In my own way I seek to find a creative interpretation of how we perceive and understand these structures and patterns. My hyperbolic sculptures use mosaic-tiling patterns and geometry to express the concepts of symmetry, infinity, 3D space and time. We perceive our world through different lenses. Art, science and mathematicians are different representations of how our world is constructed.

## 4. Illustrations

**Figures 1, 2: Infinity 1.1** *Hyperbolic Diminution* Sculpture I

**Figure 3: Time 1.2** *Hyperbolic Diminution* Sculpture I (detail)

**Figures 4, 5: 3D Wood Model 2.1** *Hyperbolic Diminution* Sculptures I – V

**Figure 6: Shapes: Pentagon and Circle 1.3** Sculpture II
**Symmetry 1.5** Sculpture II *Hyperbolic Diminution* Sculpture II
**(see also Figure 1)**

**Figure 6:** *Hyperbolic Diminution* Sculpture II (detail)

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## References


[1] D.Dunham, Artistic Patterns in Hyperbolic Geometry, *Proceedings, Bridges: Mathematical Connections in Art,Music and Science,* R.Sarhangi n (Ed.1999) pp. 239-248.
[2] Bruno Ernst, *The Magic Mirror of M.C. Escher,* Barnes & Noble, inc.1994 p.109.
[3] D.Dunham, Hyperbolic Islamic Patterns_A Beginning, *Proceedings, Bridges: Mathematical Connections in Art, Music and Science.* R.Sarhangi and Slavik Jablan, (Ed.2001) pp.247-253.
[4] B.Grünbaum and G.C.Shephard, Interlace Patterns in Islamic and Moorish Art,*Leonardo*,Vol.25.pp331-339,1992
[5] Silvio Levy, Automatic Generation of Hyperbolic Tiling,*Leonard,* Vol. 25, p.350. 1992.
[6] Reza Sarhangi, The Circle: A Paradigm for Paradox. *Proceedings, Bridges: Mathematical Connections in Art, Music and Science,* R. Sarhangi (Ed.1998) p.99.
[7] Miles Danby, *Moorish Styles,* Phaidon Press Limited, London, 1995 p.93
[8] Miles Danby op cit.

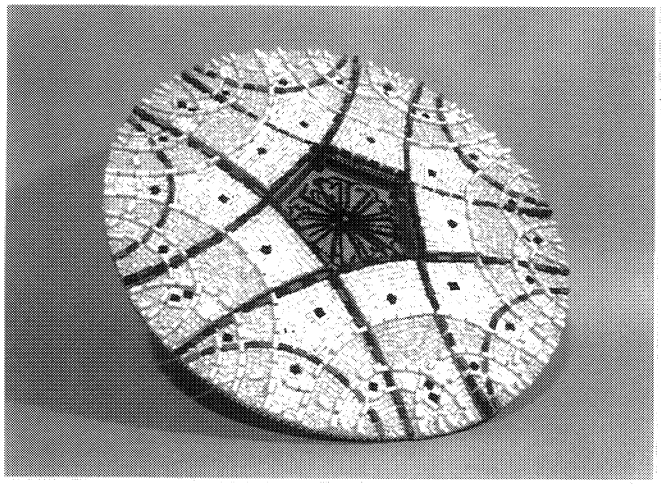

**Figure 1: Infinity 1.1** *Hyperbolic Diminution* Sculpture I

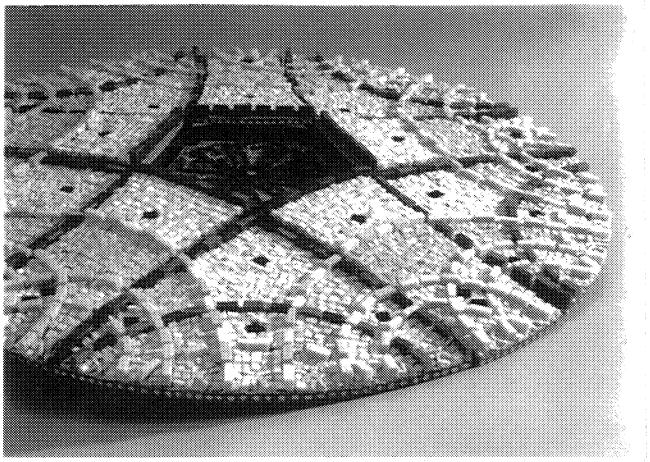

**Figure 2: Infinity 1.1** *Hyperbolic Diminution* Sculpture I

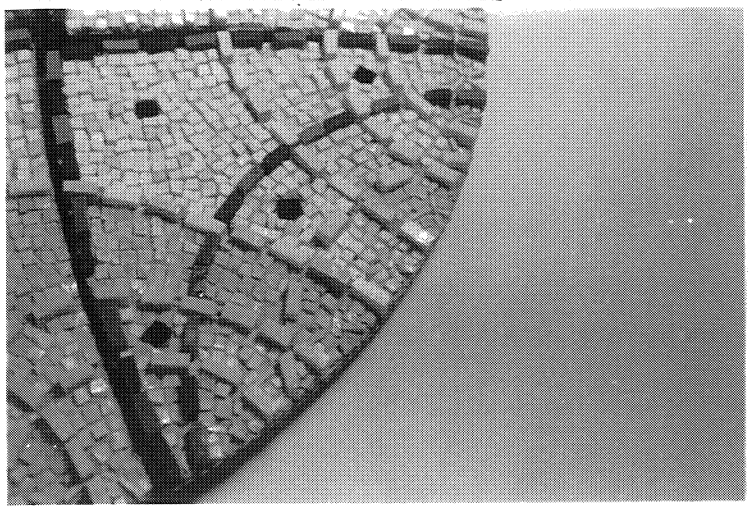

**Figure 3: Time 1.2** *Hyperbolic Diminution* (detail)

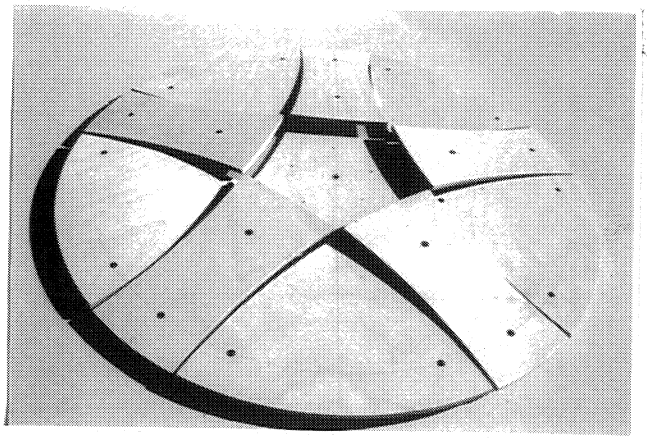

**Figure 4: 3D Wood Model 2.1** *Hyperbolic Diminution* Sculpture I,II,III,IV,V

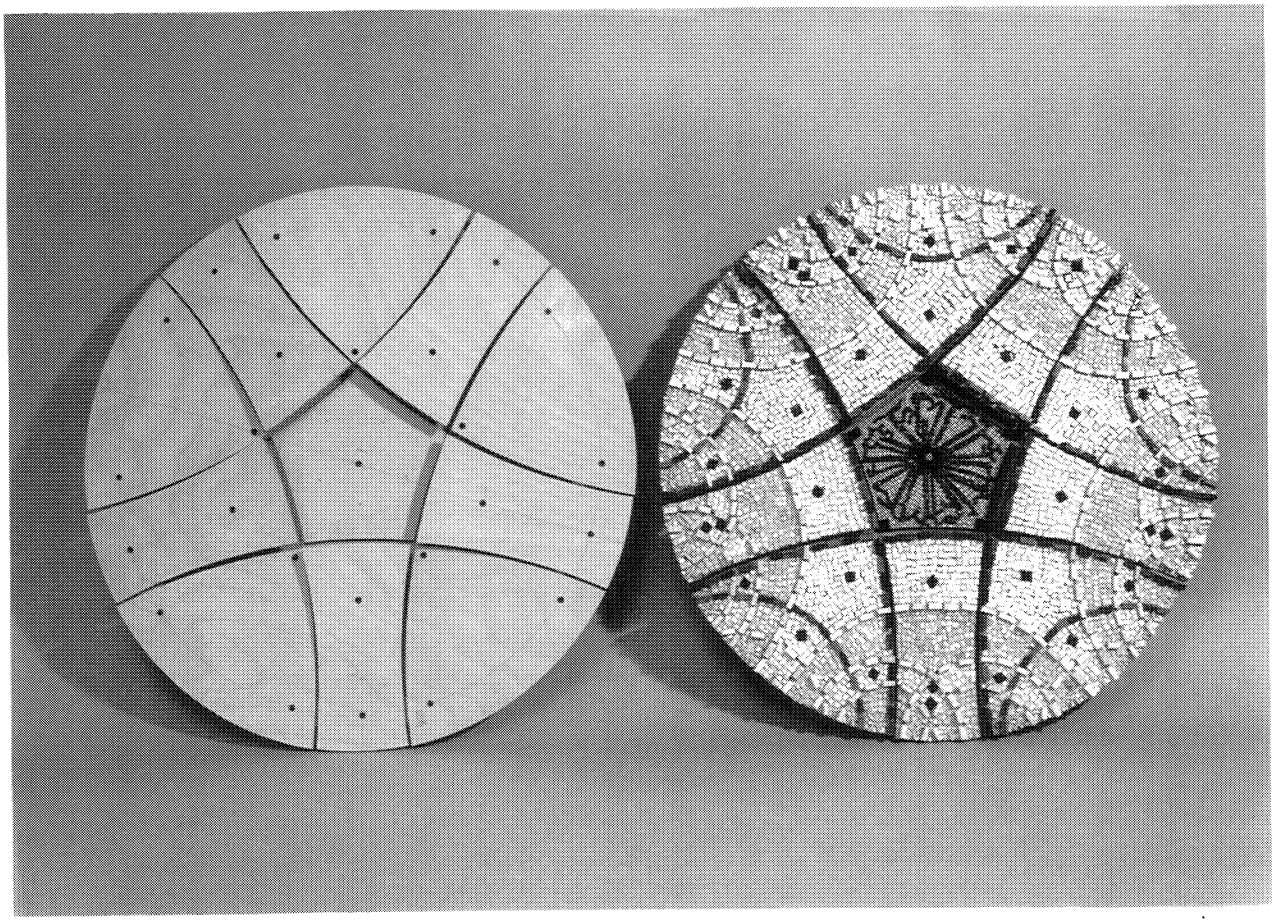

**Figure 5: 3D Wood Model 2.1** *Hyperbolic Diminution* Sculpture I

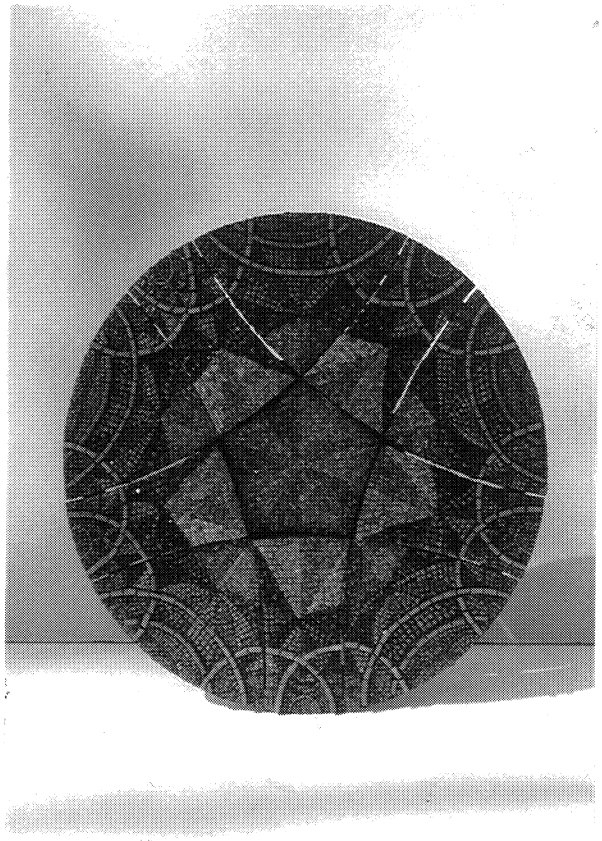

**Figure 6: Pentagon and Circle 1.3** *Hyperbolic Diminution* Sculpture I, II
**Symmetry 1.5** *Hyperbolic Diminution* Sculpture I, II
**(See also Figure 1)**

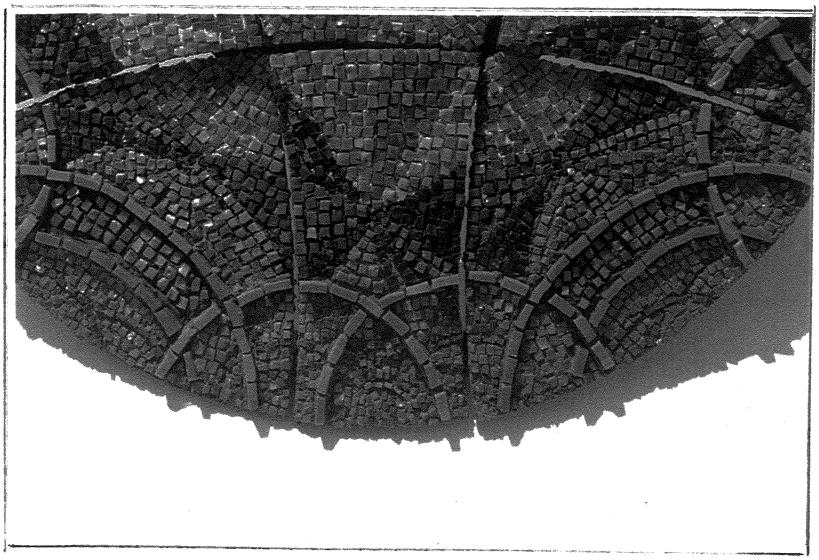

**Figure 6:** *Hyperbolic Diminution* Sculpture II (detail)