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Fractal Geometry Describes the Beauty of Infinity in Nature

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Abstract

Through the study of fractal geometry and Bionic we have been able to find out the expression of infinite in the nature structure, and to realize that there is an internal order, which is a law. A law whose expression is also the infinite. This discovery has allowed us to study Art history from a more global perspective. We have been able to see that almost every cultural manifestation has the expression of infinite as its goal. We have conclude that the searching for the infinite is what artists have called beauty and have found as something unreachable. Fractal geometry open new ways and gives us the tools to propose a new beauty canon to answer to the cultural and artistic needs of the XXI century.

1. Introduction

There have been many artistic styles along history and across different cultural moments. All of them, having beauty as a goal, used the available tools to achieve it. Nature was the beauty inspiration source, and as a result, we inherited great artistic expressions all over the world. Nevertheless, the XX Century made a turn. Beauty stopped being important to art. Art started to be a way to express sensations, emotions, even the ugly, getting in this way very far from the reality and the natural world. This fact has lead the artistic world to some state of existential crisis. While beauty used to take us to some transcendent 'place', now we are caught in a finite world that suffocate us [1].

An anthropological study of man leads us to understand that everything has its origin in nature and everything has its aim in man. Today we could say that everything that surround us is design. Therefore, a relation between nature and design must be established, as well as a relation between design and man. The bionic science allows us to do so.

The bionic science is the discipline that studies the artificial technological utility of the obtained results by the biological natural evolution. It has a common field with the artificial intelligence. In general, the main task of bionic is the analysis of the biological structures, processes, and of its technical synthesis and apply them to design optimal systems and objects for man. The starting point of the bionic is the assumption of the validity of biological evolution, co-evolution, and self organization [2]. Life techniques are optimized and, although they have individual roles, they make a coherent team. In natural life, those complex systems that are able to self organized themselves, survive. Others do not because they are rejected as a natural selection process. Therefore, natural evolution results in optimal and beautiful solutions, which inspire the artist.

Bionic is a complex multidisciplinary science, which could be seen as 'hinge-science'. It allows us to relate the natural and the human worlds, being a very useful tool for those having the responsibility to design the human habitat [3]. To apply this science we need the appropriate geometry: the fractal geometry, which describes nature in a much real way than Euclidean or other recently proposed geometries [8].

Our research, a detailed study of nature through fractal geometry and the application of bionic, has lead us to discover the expression of infinite in the structure of nature and its internal order, which is a law. A law whose expression is also the infinite. This discovery has allowed us to study art history from a more global perspective, and to conclude that every culture have always tried to express that infinite, that something unreachable which has been commonly called beauty. We have been able to study the infinite law and its development in nature, thanks to the Bionic science, concluding that everything in nature follows what we call the infinite law.

This new vision of the real world suggests us to propose a new reformulation of the classic canon, after 5000 years of no changes, to answer properly to the design and cultural needs of XXI Century. Fractal geometry contributes with the right mathematics for this proposal.

The article first explains the foundations of the natural law which has expressed the infinite through history, and its attempt to reach it as beauty expression, to conclude with a proposal to define a new way to achieve it, through fractal geometry and new technologies.

2. The infinite

2.1 Infinite in Mathematics. Man is a finite, limited being, living in the earth, also limited and finite. A finite being who talks about infinite because he nees to understand its transcendence. Mankind has studied the infinite through mathematics, using basic definitions as iteration, comparison and classification. Iteration and comparison reach two different concepts of infinite: the 'potential' infinite and the 'current' infinite [4].

The iteration process is the primary intuition of the 'potential' infinite: that which never ends because there is always more... Infinite attraction is powerful. Henri Poincaré (1854-1912) said that mathematical creation was a task about infinite, while David Hilbert (1862-1943) wrote that the mathematical analysis was nothing but an infinite symphony.

The 'current' infinite, on the other hand, originated in a geometrical context, provides solutions to quantify and solve real world problems, and includes ideal elements (infinite number, infinite point, infinite element series). Blaise Pascal fills the gap between the 'potential' and 'current' infinite as follows: "We know that there is an infinite and we ignore its nature. We know as well that numbers are infinite, and that it is true that there is a numerical infinite".

Up to XIX century, the classic continuous functions shown a particular regularity: indefinitely derivable, they had a tangent in each point. No longer; there are continuous functions which are not derivable in any point, or which fill a complete square, as the Peano's curve (1890) (see Figure 1). To work with these "teratological" functions it is necessary to work with the 'potential' infinite, shown in the iteration process. Using an indefinite iteration, Peano built a continuous curve which is not derivable in any of its points and which completely covers the 2^n dimension plane 'in the infinite'. Another example is the Helge von Koch curve (see Figure 2), which close a finite area although its length is infinite. We could say that, with this curve, we have an infinite inside a finite. Although it was published in 1906, it

has become one of the first and representative examples of fractal curve, because its fractal dimensionality. We see then that we have a geometry which allows to see the infinite within the finite, the fractal geometry (see Figure 3) [5]. It is similar to the definition of cosmos, and to the definition of nature. What is a tree? We could see it as infinite leafs within a finite volume.
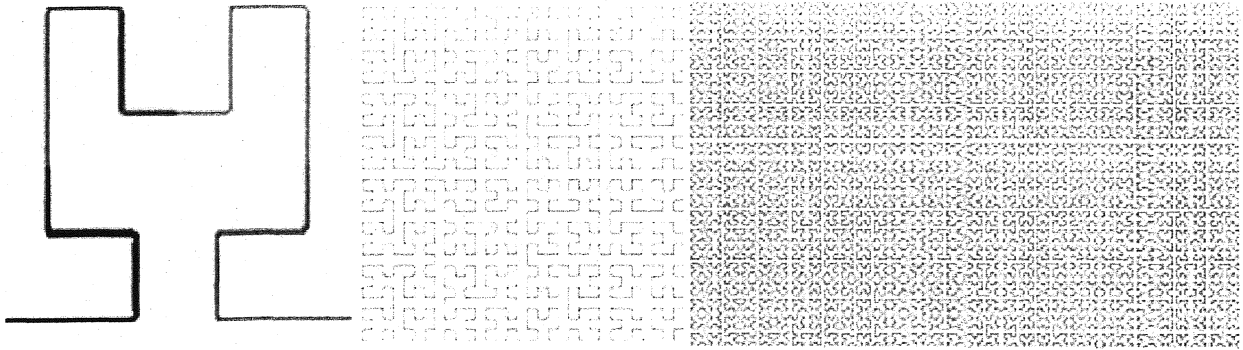
Figure 1: *Peano's curve.*
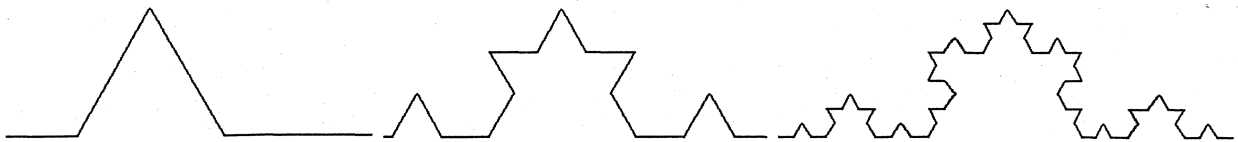
Figure 2: *Koch curve.*
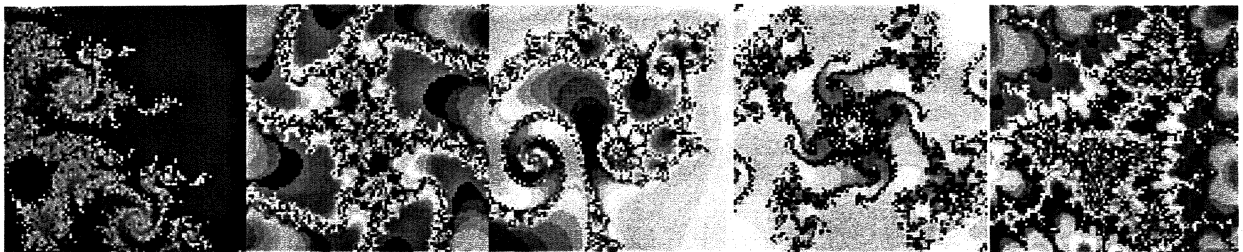
Figure 3: *Fractal developments.*

2.2. Infinite in Nature. We cannot find 'current' infinite in nature. As man is part of that nature it could be seen that he would not be able to control it. We will have to admit the 'potential' infinite, that of the result of the iteration process of generating, step by step, the points of a line, without saying that those points are in number, an infinite.

In natural world, if we observe the structure of nature we discover the 'potential' infinite through the bionic science. It shows that there is an important and relevant relation between form, matter and scale in the Universe, which is the Proportion. On the other hand, mankind has been searching very eagerly to find that 'something' which makes Universe things so beautiful. Geometers, physics, artists and philosophers thought of Beauty as a matter of proportions.

By observing nature, we discover that there are not big things or small things. Size or form are not important, what really matters is the law. A tree is just a plant which grows following a law. The category (big or small, or shape) is given by man.

X ray techniques allows us to see the reality from a different perspective. This makes our knowledge and sensibility bigger. We can enjoy forms that are not available to our sight and we discover that all

those forms have something in common, an order structure. This structure is a common law to all life being and empowers them to develop themselves and to work in a specific manner.

To understand better this issue, lets analyze the structure of a bird wing. If we cut the wing, we observe the way that the feathers are joined to the bond. We see that the inferior side is different to the superior one, it is composed by a central structure and another perimeter structure, filled with a microstructure. If we see in detail the small part in scale, we will see that it is composed of bars and barburillas, which are with the teeth and bars, the lineal elements. If we make a section of the bars and barburillas we see that, inside each barburilla, the sum of all the micro airs that are inside the feather is what make the bird light and able to fly. One of those barburillas, with its very small teeth which join the valve that makes the feather, is made up by 0,04 millimeters, also covered by a 0,02 mm wall. This structure is similar to the bonds. The birds bonds are very resistant to twisting: if all that bond matter of birds were just un solid bond, the bond will break. In nature, forms are not important, not even the way matter are placed. What happens is what we could called multi fraction of the effort: many little elements joined together, achieve that, what is week as one unity, gets super strong when millions of those small unities work together. This fact happens in every life being. If use special instruments, we see that what is big and solid to our eyes, in fact is an infinite world of elements with self similar forms, included into other self similar forms, and so on and so forth, up to the end finite of matter, up to the molecule.
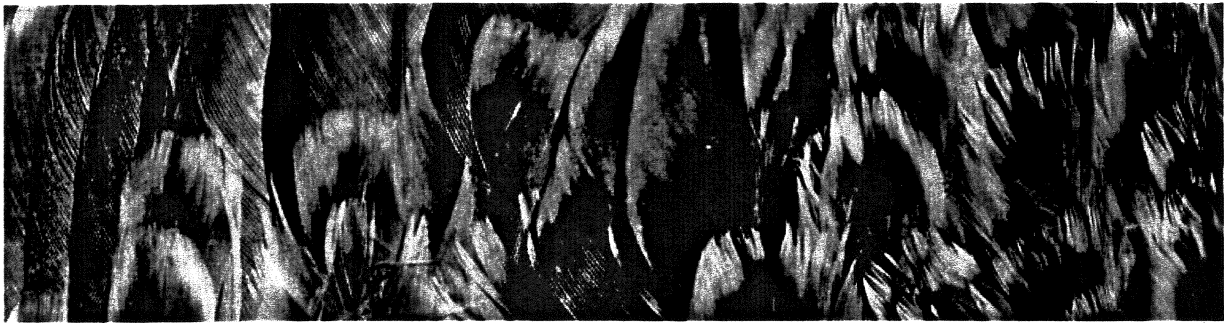
Figure 4: *Bird wing.*

The law discovered under the X Ray talks to us about iteration, self organization and, as we explained in the first section, shows us the 'potential' infinite. Therefore, we can conclude that the formal expression of the law is the 'potential' infinite.

2.3. Infinite in Art. Along History, artists have been searching for beauty, they have tried to discover the universe's laws. Laws which, as we shown above, are the ones that express infinite.

The expression of infinite has been key in architecture, painting and the rest of the arts. In some cases, that expression was evident, as the gothic style; in some others, more structural, as the ancient Greek art. As art is expression, depending of the cultural understanding of infinite and its manifestation, at every époque we see ideas either in relation with natural and real world, or on the contrary, far from it.

That is why the concept of infinite has played a key role in the understanding of man. The aesthetic of the limited, in contraposition to the unlimited or infinite, has produced the fundamental transformations that man has expressed in his artistic representations. The finite is a limit, while the infinite transcends...

In Granada we have the example of Muslim architecture, where we find that natural infinite expressed in its walls, colors and textures. Looking at the Muslim architectural texture we observe that the objects surface might have a transcendental quality due to the material richness and its models. An example of material richness is the grained marble. The models are achieved through geometric patterns which

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interconnect forms and colors. The texture might help the substance to loose its heavy appearance, to posses the desired spiritual quality, and, as a consequence, to transform the surface making it immaterial and infinite. All these, combined with the surrounding vegetation and its light effects, results in a perception of infinite in the place (see Figure 5).
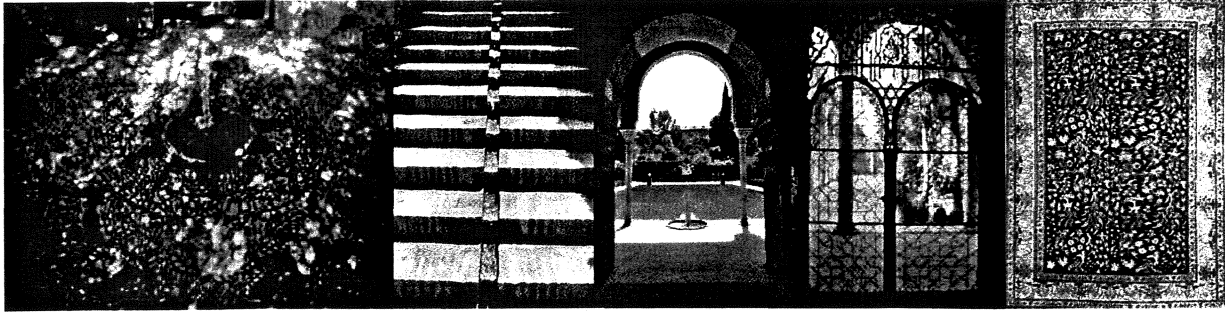
Figure 5: Muslim Art.

As we have done with Mulsim Art, we could do the same across countries, cultures and époques to find that natural infinite expressed in art. With one exception, the XX century, when a rupture with reality was assumed, in both ideas and material expression of natural laws.

2.4. Infinite in artifacts. Artifacts are buildings, objects and elements where man develops the totality of its vital activities. As they are bio-physiological interfaces, their design must subordinate human needs requirements. The responsible for a good construction of these artifacts are engineers, designers and architects.

Man is a natural and biological system, with many elements and functions. It is a very complex being, characterized by an extraordinary adaptability capacity, but under several needs which could be classified into material and spiritual needs. Infinite is important to design artifacts to satisfy both.

Our research shows that it is possible to find a design law for human artifacts that achieves aesthetic beauty and appropriate functionality. Nature, as a slow evolution result, achieves optimal system structure solutions; it works following some laws which are optimal for its own development and that are in a continuous change to reach the perfect equilibrium state that constantly tends to break. Those laws have the 'potential' infinite inherent in its generation.

It is precisely in the universal harmony laws where nature, aesthetics and functionality are fusioned into a unique role which is the expression of infinite. This role is found in a proportion which has been specially considered along history: the golden mean. There is a big change from studying the golden mean with the Euclidean geometry (nature as a model) to study it through fractal geometry (nature as a system). The current mathematical tools allow us to study that proportion in order to discover the key to define the natural order.

3. Infinite as beauty generator

The scope of this paper does not allow us to present a detailed study of the fractal geometry, but we can point out that there is an important element that acts as a connection, or as a continuity, between the classic theories about beauty and the beauty that we need nowdays for the culture and environment of XXI century: the golden mean. Its study, through fractal geometry will help to open new ways to express beauty. These new ways will not propose forms, but the laws of the nature structure.

3.1. Fractal geometry and the laws. A fractal is a law, by which we cloud create almost all the biological species. The invention of fractal geometry changed the way to understand our world. Now we study life beings with laws, not with forms.

If we observe nature with laws we will see and understand curious things, as the obsession of nature for spirals. The spiral is the geometrical figure that presents almost every life being (see Figure 4).
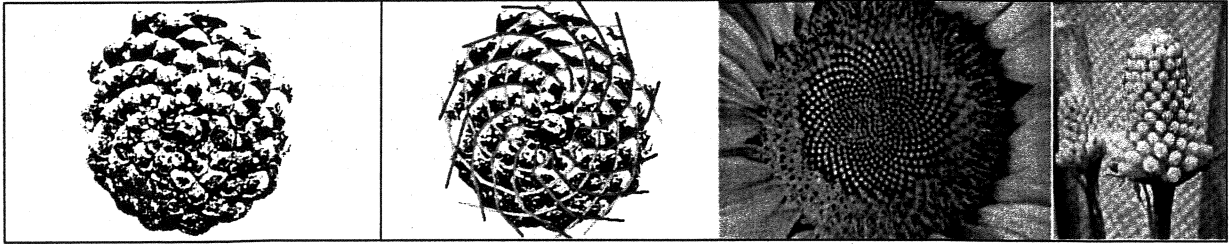
Figure 4: *Spirals: pineapple, sunflower, flower.*

If we look at a pineapple, we see that the pines are organized as a spiral. If we make an spiral diagram, we can check that different spirals families coincide: they are different but they all get through common points. Putting them all together, we see how they are and what patterns they follow. It is interesting to see that spirals correspond to the golden mean formulation and the golden mean is a fractal [7].

3.2. The golden mean: a paradigm of self organization and self similarity. The golden mean is the limit of one of the most important numerical systems in architecture and art and most spread in biological criatures: the fibbonaci sequence. Nevertheless, the golden mean is also the result of several iterative process, which proceed in an autonomous and automatic way. Therefore, we could identify in the golden mean a paradigm of that typical dynamic order of the self organization, as well as that form of symmtry which self organization usually has: scale simmetry or self simmetry (by which, using the local zoom into a structure, what we find is identical to itself). Therefore, the golden mean is a special fractal type. We should remember that the euclidian geometry is just a particular case of the more general geometry, the fractal geometry.

From the above, we can say that the way nature self organizes could be defined through the self similar fractal mathematical sets. This discovery has determined a cultural mutation in our way to conceive and perceive the natural structure dynamics and genesis. A self similar structure: similar to each part which is a copy of the structure and, as a consequence, of the entire structure. This happens in the natural organism, that is why fractals are the appropriate tool to formulate the a cannon and, therefore, be able to create an expression of beauty, wider than the classics´.

Professor Caglioti, nuclear physic of Milan Politechnical School says: "The sense of the beautiful constitutes an ethical criterion, of excellence, of survival of the species. Furthermore, the sense of the beautiful constitutes an aesthetic criterion, of beauty and truth, of the survival of ideas and scientific theories, as well as of works of ingenuity: architectural or musical structures, the visual arts, sculpture and literary works. The sense of the beautiful constitutes a spur to creativity and a selection filter in the planning of a handicraft or in the survival of a technological or mass produced product" [6]. Because beauty is the inherent order to nature and carries the 'potential' infinite.

Since Platon to nowadays, scientific reseach instruments have became very precise. The knowledge field has increased a lot. And the way of working and communicate has changed. As well as the way of

human interaction. Our vision of the world has been widened and some sectors have been developed in a very fast and disconcerted way.

Nevertheless, in every critical situation in which to make a decission is needed, the sense of the beautiful, as a law, assumes an important neuralgical role in the election orientation. The neuralgical moment of every evolutive process, that is, the moment in which a system decides in its interior, in front of an initial fluctuation, whether to go back or forward to produce a new structure –and therefore, might be called mutation or invention- is the selection moment. This critical moment is controlled by the sense of the beautiful. The beautiful tends to appear as a result of a natural evolutive process which is generated naturally.

In nature, every criature, once it is conceived, develops itself autonomously, without any hiperburocratic process impossing one direction to the millions of millions of millions of atoms which conforms it, to be in a specific possition in a precise moment. Just take a close look to nature to see that the evolutive processes are characterized by the self organization. In all the cases, due to the systematic self organized process, suggesttive self similar structures are created; all of them characterized by the fact that, using the local zoom, the entire structure is found. This is the base of fractal geometry.

If we find beauty in the structure of nature, its expression is the infinite, and the fractal geometry is the scientific base to extract that structure, in order to apply it to a variety of fields, as many as sciences using the natural laws, including design and human creativity. In particular, to apply it to architecture, being able to develop new artistic forms which guarantee an adequacy to the enviroment and a better functionality, and that answer to both our material and spiritual need of infinite.

Fractal geometry is therefore the geometry of nature which expression is the infinte.

4. Conclusion

We have carried out a study of the internal structure of nature through the bionic science and the fractal geometry, coming up with the following conclusions.

We have discovered the expression of infinite as the internal natural order. This order is what we call beauty. And beauty has been the desire of art along history.

That order, beauty, is the same law that nature uses to achieve optimal system structure solutions. As the result to desing using that order, natural, optimal, environmental and beautiful artifacts are achieved.

This study sets the basis for the definition of a new canon for design. The innovation of the canon is that it is based on laws, not on forms. It is the first proposal in 5000 years and has been made possible due to the new technologies, computers and new sciences and geometries.

This new canon will be able to open up new artistic ways that will answer to the cultural and environmental needs of the XXI Century.

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