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The Mathematics of Quilting: A Quilter's Tacit Knowledge of Symmetry, Tiling, and Group Theory

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Abstract

This paper discusses the mathematics of quilting—particularly symmetry, tilings, and group theory. Using Michael Polanyi's theory of tacit knowledge, it furthermore suggests how an average quilter has indwelt knowledge of these explicit forms of mathematics.

For over 20,000 years women have been spinning, weaving, and sewing [1]. Traditionally, these art forms have been identified as being appropriate for small-minded, domestic, and dutiful women whose goal in life is to serve their families—and certainly not to excel them in academic endeavors. Likewise, skills like mathematics have been portrayed throughout the ages as men's work; appropriate for vast-minded, undomesticated, and assertive men whose goal is to serve the development of man's intellect. As Claudia Henrion states, "...mathematics has traditionally been identified with the realm of mind, while women are traditionally associated with bodies, children, hearth, and home.... The two spheres were seen not only as separate, but also as hierarchically ordered: the life of the mind was considered far more important than the life of the home [2]." For this reason, mathematical skill and women's work in the handicrafts have often been portrayed as being mutually exclusive. In this paper I would like to suggest however, that this is not the case; mathematical skill and women's arts are not mutually exclusive. In fact, mathematics is an essential tool used in many of these art forms. Handicrafts, and in particular quilting, require amazing mathematical knowledge and skill, which until recently has gone unnoticed. Quilting requires a working knowledge of everything from basic arithmetic to the complexities of geometrical and symmetrical analysis—including symmetry, group theory, and tiling. Yet for the average quilter, only surface level mathematics is comprehended consciously. Using Michael Polanyi's theory of personal knowledge, I would furthermore like to suggest that the average quilter has a tacit knowledge of such mathematics-in which they know and practice the inner-workings of symmetry, tiling, and group theory without explicitly knowing. In a very real sense, quilters have always been, and continue to be, quite sophisticated mathematicians-whether or not they recognize themselves as being such.

Personal Knowledge

Michael Polanyi suggests in his Theory of Personal Knowledge that knowing is an art form in which the knower understands infinitely more than she can articulate. Called 'tacit knowledge', this ability in humans to comprehend external facts without being aware of them specifically, accounts for our ability to function in the world. "...tacit knowledge forms an indispensable part of all knowledge [3]," and it is this part of knowledge which allows us to process meaning, to produce an outcome, or reach a goal. This telos, to which we hone our awareness, Polanyi calls *focal awareness*. Consider the following example:

"When we use a hammer to drive in a nail, we attend to both nail and hammer, *but in a different way.* We *watch* the effect of our strokes on the nail and try to wield the hammer so as to hit the nail most effectively. When we bring down the hammer we do not feel that its handle has struck our palm but that its head has struck the nail. Yet in a sense we are certainly alert to the feelings in our palm and the fingers that hold the hammer. They guide us in handling it effectively, and the degree of attention that we give to the nail is given to the same extent but in a different way to these feelings. The difference may be stated by saying that the latter are not, like the nail, objects of our attention, but instruments of it. They are not watched in themselves; we watch something else while keeping intensely aware of them. I have a *subsidiary awareness* of the feeling in the palm of my hand which is merged into my *focal awareness* of my driving the nail [4]."

Tacit knowledge, which is subsidiary awareness, functions so as to allow us to complete a task at hand, which demands focal awareness. The relationship between subsidiary awareness, which we attend 'from', and the focal target, which we attend 'to' Polanyi calls *functional relation*, or a *from-to relation* [5]. Thus, tacit knowledge embodies three key constituents, all of which relate to one another: "...first, the subsidiary particulars; second, the focal target; and third, the knower who links the first to the second [6]." In an act of tacit knowing, that which we attend 'from' (i.e. the subsidiaries), to that which we attend 'to' (i.e. the focal target) are more easily called the *proximal* term and the *distal* term. Of the two, "it is the proximal term...of which we have a knowledge that we may not be able to tell [7]." Though completely assimilated into our knowing, these subsidiaries are nonetheless unspecifiable.

How is it that we are not able to articulate this tacit knowing? Polanyi speaks of tacit knowledge ontologically, as an indwelling of our being—as *personal* knowledge. "We pour ourselves out into (our subsidiaries) and assimilate them as parts of our own existence. We accept them existentially by dwelling in them [8]." For this reason subsidiaries are essentially unspecifiable. Our knowledge of them lies in our very being. Thus,

"...we cannot learn to keep our balance on a bicycle by trying to follow the explicit rule that, to compensate for an imbalance, we must force our bicycle into a curve—away from the direction of the imbalance—whose radius is proportional to the square of the bicycle's velocity over the angle of imbalance. Such knowledge is totally ineffectual unless it is known tacitly, that is, unless it is known subsidiarily—unless it is simply dwelt in [9]."

And riding a bicycle is similar to any other task. For in these tasks we embody, and thus know tacitly, the complexities of each task without knowing them explicitly.

Women's handicrafts, furthermore, also involve immense indwelt knowledge, which is not necessarily explicitly known or spoken. This knowledge, like bicycling, demonstrates itself as we act upon focal targets. In her book about mathematics and handicrafts, Marry Harris demonstrates this well. She quotes Gordon Sutton [10], "Already by 1846 one Inspector had reported of the British and Foreign Society's girls' schools that 'it may be said that the instruction of the majority... is not so much in reading, writing and arithmetic, as in reading, writing and needlework' [11]." Instructed in the areas of handicrafts, these young girls were denied a proper education in mathematics, as it would cause 'too much mental strain' [12]. Ironically however, Harris goes on to explain, "...much of the girls' work had been mathematical all along. Every medieval embroidery required attention to symmetry and structure....Every garment structured from flat cloth of particular dimensions to fit the three dimensional form of a particular body, is an exercise in practical geometry [13]." As a result, these young girls acquired knowledge in mathematics, subsidiarily as the case may be, and embodied this knowledge in their ability to sew. As Polanyi states, "By watching the master and emulating his (or her) efforts in the presence of his (or her) examples, the apprentice unconsciously picks up the rules of the art, including those which are not explicitly known to the master himself (or herself) [14]." Though never formally trained in geometry, symmetry, or the complexities of multi-dimensioned planes, these girls acquired a tacit knowledge of these mathematical skills, and exemplified their knowledge of such through their work.

The history of patchwork quilting yields much the same story. In late eighteenth-century America, patchwork designs began to appear, and slowly, over time began to grow in reputation. "The art of patchwork quilting flourished in the period 1775-1875, and the designs (and especially the naming of the designs) were inspired by many events, such as the admission of new states, the opening of the West, the railroad, and many political and social movements [15]." Designs like "Road to California", "Railroad Crossing", "Drunkard's Path" and "Aircraft", which commemorated events both large and small in the lives of working American



women, were sewn into history with the use of geometric figures and symmetries. Yet these designs were not created by astute mathematicians. Rather, they were designed by the minds of women with little education and few resources. As was often the case, "...the country needlewoman depended largely upon her imagination for her masterful designs [20]." From these initial patterns however, new designs emerged as women shared their ideas with one another. Around 1800 the "9-patch" design and "Grandmother's basket" appeared. In 1806 the "Irish Chain" pattern is known to have existed, possibly being the first use of the block pattern as an overall quilt design. By 1815 eight-pointed stars of all sorts, including the "Ohio Star", were being circulated and reproduced [21], and between 1834 and 1859 "LeMoyne Star", the "Texas Star", and the "Blazing Star" were all in development. All of these patterns are beautiful and artistic designs, but they are so *because* of their mathematical structure. Thus despite





Blazing Star [23]

the inability of the women who created these designs to recognize their use of such math, they did in fact have some awareness of the mathematics that went into their patterns. By 1840 then, when block patterns of patchwork design had increased tremendously in popularity, this new craze in quilting had brought to the working women of America new mathematical knowledge never before assessable to them [24]. The types of mathematics that women came to use and therefore tacitly understand through patchwork quilting include symmetry, tiling, linear algebra, and subsequently geometry, fractions, and arithmetic.

Symmetry and Quilting

Traditional quilt designs are typically based on symmetry groups, and the women who create and recreate them understand their complexities thoroughly, though tacitly. Consider the 17 symmetries of a plane. In traditional patchwork quilting, at least 12 of the 17 symmetries appear in the formation of designs and patterns.

Of these symmetries, the symmetry P4M is by far the most utilized symmetry in traditional quilting [25]. I believe this to be true because P4M has a "simple complexity" that appeals to quilters. Its

simplicity is pleasing to the eye, as many of the simpler symmetries are, such as PM, P4, or PG symmetries, and does not appear too chaotic, as can be the case with symmetries groups like P3M1 or P4MG. In addition, and very importantly, P4M symmetry lends its self well to the piecing process. With this symmetry, the 90° rotations and four mirror lines make piecing easier for stars, flowers, and other designs typically found in quilting simply because the piecing happens directly on those mirror lines. Coming together at one single point, the 4 rotated quarters that create this design are made up of a smaller, asymmetrical unit—a right triangle. This right triangle has mirror lines along two edges, and "...eight of these unit triangles, four plain and four mirrored or reversed, will make up the traditional block [26]." The following design, a "LeMoyne Star" in a "Feathered Star", was made by my grandmother, Ella Huyler, and demonstrates this symmetry well.



"LeMoyne Star" in a "Feathered Star" by Ella Huyler, 1998. Lines have been overlaid along the mirror lines for visual effect.

The quilter who designed this pattern would have recognized the simple elegance and wellbalanced beauty created by this four mirror-lined symmetry *because* its aesthetic characteristics are so well utilized in this design. Consequentially, creator of this design was tacitly aware of P4M symmetry. Yet any quilter piecing this design must also be tacitly aware of P4M symmetry. For as a quilter cuts out the eight identical diamonds needed to make a LeMoyne star, and then pieces them together—first along the diagonal mirror line, and then along the horizontal and vertical mirror lines (as is indicated in the illustration), a sense of the importance of those mirror lines and their relationship to the overall design begins to formulate, though tacitly, in a quilter's mind. Without such knowledge a poorly designed and visually flawed pattern would emerge.

In the design "Buck-Eye Beauty" this is also the case. The following rendition of "Buck-Eye Beauty" has C2M symmetry which also involves both rotations and mirror lines. In this design, the "original figure" is rotated 180° to make the unit block. Next, the unit block is reflected across mirror lines running horizontal and vertical, shown as arrows. In effect, the overall design appears to have staggered sets of diamonds.



"Buck-Eye Beauty" by Katrina Hebb, 2002

Based on Polanyi's theory, a quilter piecing this design would understand C2M symmetry tacitly. This is evidenced by the fact that C2M symmetry is literally sewn into this pattern, step-by-step, throughout the piecing process—for the most economical way of piecing this design is to follow the order of how C2M symmetry functions. First, the pieces, the four-patches and the two-triangle squares, which make up each of the "original figures", must be sewn together. Next, each four-patch is sewn to a two-triangle square, creating the "original figures". Once this task is complete, the blocks are sewn together after rotating one of the "original figures" 180° from the original fixed position of the figure. When all of these now unit blocks are complete, the quilter would then spend time assessing color values of each staggered diamond [27]. This is possible only if the quilter positions each unit block so that all of the blocks line up as mirror images. If the blocks are not aligned properly, as mirror images, the "staggered diamonds" do not appear and the quilter cannot assess the placement of color values. In this respect, any quilter piecing this design would, by the very nature of the piecing process, come to tacitly understand C2M symmetry.

Quilters also possess tacit knowledge of symmetry in other ways. I myself am a case in point. As a novice quilter it took making mistakes on my first few projects to realize that non-regular finite shapes do not have rotational symmetry. Until having done research for this paper, I could not have explicitly told you that I knew this. It was simply tacit knowledge that I began to understand and assimilate as I pieced together the non-regular shapes that make up the following two projects.





Excerpt from "Storm at Sea", by Katrina Hebb, 2002 The eight-pointed star contains non-regular triangles.

"Summer Stars", by Katrina Hebb, 2002 The parallelograms in these eight-pointed stars are non-regular.

As I gained more experience however, I began to apply this piece of knowledge while using templates to cut out shapes without even realizing what it meant. In effect, that I learned about the properties of non-regular shapes by way of piecing [28], and integrated this knowledge as I became a more proficient quilter, suggests that I tacitly knew that rotational symmetry does not lend itself to non-regular shapes.

Quilts and Tilings

Quilting also falls under the mathematical category of a tiling, and many different tilings occur in quilting—including regular and periodic tilings. Just as with symmetry, the art of tiling becomes tacit knowledge of quilters as they use tiling techniques in their quilts. To begin with, it must be stated that quilters of any ability understand tacitly the concept of tiling because its definition underlies the whole of quilting—quilts *are* a collection of disjoint open sets that cover a plane. And although quilters might not understand explicitly this definition, they most definitely understand how the rules of tiling function: no gaps and no overlaps are allowed in patchwork quilting.

Regular Tilings

Regular tilings are made of a single regular polygon and follow the rule that all vertices must have the same configuration, in addition to the general rule of tilings, which states that all vertices must have

interior angle sums of 360°. Of the many regular polygons that exist, only three of these special polygons are capable of tiling a plane regularly. They include the square, the equilateral triangle, and the hexagon. Pentagons, for example, do not function properly in a regular tiling because at any given vertex they do not have an interior angle sum of 360°.



Equilateral Triangle



Pentagons [29]

Many quilt designs exist that utilize all three regular polygons in regular tilings. In the following example, pieced by my great-great grandmother Susan Smith, the quilt design, called "Trip Around the World", employees the square as its prototile in this monohedral tiling [30].



Made by Susan Smith, circa 1930

That my great-great grandmother created this quilt using only 2-inch square blocks [31] indicates that at some level she was tacitly aware of regular tilings of the plane—for this quilt was made by following explicitly the rules of regular tilings.

Other examples of regular tilings in patchwork quilt designs include "Thousand Pyramids", which uses equilateral triangles, and "Grandmother's Flower Garden", which utilizes the hexagon as its prototile.



"Thousand Pyramids" [32]



"Grandmother's Flower Garden" [33]

That quilters throughout the years have utilized every regular tiling pattern in these designs, and have varied them by means of color symmetry to create many, many other exquisite tiled patterns indicates that

collectively they posses a tacit knowledge of the nature of regular tile patterns, despite their lack of conscious awareness.

Periodic Tilings

A periodic tiling is a tiling that contains at least two translations in non-parallel directions. Though the bulk of traditional quilt patterns cannot be classified as periodic tilings (this owes to the fact that the majority of traditional quilt blocks work with mirror symmetry rather than translation symmetry), the majority of quilts *themselves* function as periodic tilings. This is based on the layout of most patchwork quilts. Traditionally, patchwork quilts are *set*, which is to say, they are constructed from a number of similar blocks and are laid out according to translation symmetry. There are various setting styles, including straight set, alternate straight set, diagonal set, and lattice set [34]. A *straight set simply* translates the quilt block. An *alternating straight set* separates the quilt blocks and a *set block*—a block of plain fabric that separates the quilt blocks. A *lattice set* separates the quilt block with the use of a lattice, and a *diagonal set* uses both quilt blocks and set blocks, like the alternating straight set, except that all blocks in the diagonal set are rotated 45° and set with corner and side triangles to keep the quilt squared. The following three pictures illustrate a straight set, an alternating straight set, and a lattice set.



In all of these examples the quilts *are* periodic tilings, and any quilter piecing these quilts would come to understand the periodicy of their quilt tacitly as they pieced the quilt blocks according to translation symmetry.

Group Theory and Quilting

Group theory is quite literally the study of symmetry. By definition, "a group is an ordered pair (G,*), with the following properties:

- 1) Closure under the given operation: $\forall x, y \in G, x^*y \in G$
- 2) Associative Law: $\forall x, y, z \in G, (x^*y)^*z = x^*(y^*z)$
- 3) Identity Element: $\exists e \in G$ such that $\forall x \in G$. $x^*e = e^*x = x$
- 4) Inverse Elements: $\forall x \in G, \exists y \in G$ such that $x^*y = y^*x = e$.

It is quite probable to say that the majority of quilters cannot recite this definition, or even comprehend what it means when read to them. However, this does not indicate that they do not already know, tacitly, what group theory is. For quilter utilize properties of group theory when setting quilts that require rotations of blocks. For example, consider the log cabin pattern:



The log cabin block is another old and traditional patchwork quilt design and can be varied upon in numerous ways. The center square is traditionally red and symbolizes the hearth in a home. To make a log cabin block, begin with the center square and join "logs" around it in a circular fashion "...starting with shorter logs and working out to the longer ones. The logs are joined in a circular fashion with the block being rotated 90° counter-clockwise after each log is joined. The current log being joined is always started on the log which was just joined to the block [37]."

The structure of the log cabin block has rotational symmetry of order 4 (P4 Symmetry) and forms a cyclic group, which is also abelian. In addition, it is isomorphic to Z_4 under addition for mod 4 (Table 1). Thus the group formed by the log cabin block (Table 2), where the above log cabin represents R_0 , has one-to-one correspondence and is operation-preserving. This is evident when comparing the two tables.

· +	0	1	2	3	*	R ₀	R ₉₀	R ₁₈₀	R ₂₇₀
0	0	1	2	3	R ₀	R ₀	R ₉₀	R ₁₈₀	R ₂₇₀
1	1	2	3	0	R ₉₀	R ₉₀	R ₁₈₀	R ₂₇₀	R ₀
2	2	3	0	1	R ₁₈₀	R ₁₈₀	R ₂₇₀	R ₀	R ₉₀
3	3	0	1	2	R ₂₇₀	R ₂₇₀	R ₀	R ₉₀	R ₁₈₀

Table 1: Cayley table for Z_4^+ Table 2: Cayley table for Log Cabin Group

When numerous log cabin blocks are joined together, quilters use the properties of group theory, often without their even knowing. For example, when laying out a design such as in the following two examples, a quilter relies tacitly on all four of the properties of group theory to complete the task at hand. This occurrence has to do with the fact that to set a pattern using log cabins, individual blocks must be rotated precisely to create the desired effect. This often requires great forethought and endless numbers of rotations of blocks to get the exact pattern sought.



Quilt Designed and Made by Ella Huyler, 1993

"Log Cabin Blues" made by Ella Huyler, 2000

Consider the associative property. Throughout the process of setting a log cabin quilt, a quilter tacitly depends upon her knowledge that for all rotations R_0 , R_{90} , R_{180} , and R_{270} , any two rotations followed by a third rotation are equivalent to the second and third rotations followed by the first—and so forth and so on. This knowledge of the associative property, as with any tacit knowledge, comes with time and experience of rotating log cabin blocks in every which-way possible, and saves a quilter time when setting this particular quilt pattern.

In the same manner, a quilter endlessly rotating log cabin blocks would also begin to understand the concept of both inverse elements and the identity element. This occurs as a block, rotated from what might be considered the "starting" rotation—that is, the identity element—is rotated back to the identity element from a different rotational position. A quilter might speak of inverse elements as "rotating the block three turns (from R_{90}) or two turns (from R_{180}) to return to the original rotation", but their language still indicates that they understand inverse elements.

Likewise, a quilter rotating log cabin block after log cabin block would also quickly learn that any combination of two rotations will only result in one of the four possible rotations of this group, and no more. Thus, a quilter would also tacitly understand closure of the log cabin group under the rotational operation.

In this way, setting a quilt such as a log cabin quilt—a design other than a periodic tiling or of a one-patch design—would bring a quilter to understand tacitly the properties of group theory. Piecing log cabin blocks together requires exactness on the part of the quilter, and group theory is an important tool that aids in the efficiency of the setting process.

Conclusion

In conclusion, the mathematical complexity involved in both the designing and the re-creating of such geometric art is astounding—for it is the artistic implications of such mathematics that makes quilting an art. To capitalize on the ascetical value of symmetry, one must grasp, at least tacitly, a sense of translations, mirror lines, rotations, and glide lines. And to understand rotational symmetries, and therefore set a log cabin quilt, a quilter must understand, tacitly, properties of group theory. More basic even to the task of guilting, a guilter *must* tacitly understand tiling in order to sew a guilt together properly. Thus to say that because quilters cannot articulate explicitly the properties of group theory, or expound on other symmetrical characteristics of their quilts, they do not understand these branches of mathematics *cannot* imply that they do not tacitly know these forms of mathematics. As Polanyi states, "...your knowledge of it lies in this very use of it [38]." Quilters do have a tacit knowledge of mathematics, and it is evidenced in their art. In some respects even, it is plausible to consider that quilters might even have a more intricate knowledge of symmetry, tiling, and group theory than some who purport to teach it. For "...in a number of respects it is harder to work in application than in pure mathematics..."[39] In application the variables are greater in number, more complex, and their relationship to the problem at hand more complicated. In addition, the fact that mathematical concepts must be applied in quilting in an artistic fashion lends even greater depth to a quilter's relationship to Their work involves a hands-on relationship unmatched by any lecture or board mathematics. demonstration. No matter the degree, however, it is evident that quilters tacitly understand the complexities of symmetry, tiling, and group theory and apply this knowledge in very beautiful and artistic ways. In this respect women have, quite literally, always had a hand in mathematics.

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[25] In my analysis of nearly 200 patchwork quilt designs, a third of all designs had P4M symmetry.

[26] Ruth McDowell, Symmetry. Lafayette, CA: C&T Publishing. p. 11. 1994.

[27] Color value determines the best placement of corresponding regions of a design, based on

their color. In this design light-colored blocks are placed next to dark-colored blocks,

which creates the "look" of each staggered diamond.

[28]Later I learned that non-regular shapes are called "reversed patches" in quilting.

[29] both images taken from www.coolmath.com/tesspag1.htm

[30] In quilting, a monohedral tiling such as "Trip Around the World" is called a "one-patch".

[31] I have since added two borders to her regularly tiled quilt.

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