Pedagogical Principles for Teaching Art in Mathematics Courses

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Abstract

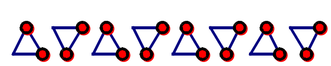
We use the one-dimensional frieze patterns to illustrate basic principles of pedagogy in teaching mathematical applications to art. Five pedagogic principles are used: (a) Creating a model of student cognitive process, (b) defining a specific problem domain, (c) developing algorithms, data structures and notations representing this model, (d) creating a teaching strategy based on the algorithms and data structures, (e) requiring problem richness. We show how student interviews facilitate the development of concepts, which are pedagogically useful in enabling students with limited mathematical proficiency to appreciate the mathematics and frieze patterns of various cultures.

1.1. Goals. This paper explores five pedagogic principles useful in facilitating the presentation of applications of mathematics to art. The methods presented were used to develop a two week module in an introductory liberal arts mathematics course taught to non-mathematics majors, some having extremely limited mathematical proficiencies. The art module showed how applying the theory of symmetry classification to cloth and pottery patterns could lead to a definition of cultural diversity [1].

1.2. Review. A set of points, $\mathbf{P} \subset \mathbf{R}^2$ is a *frieze pattern* if **P** contains a bounded subset **M**, the *motif*, such that **P** equals the union of *translates* of **M** by a fixed vector (which we assume is in a horizontal direction). A rigid motion of a pattern is a *symmetry* of the pattern if the pattern remains invariant under the rigid motion. There are five symmetries possible for frieze patterns: *translation*, *reflection in a horizontal mirror*, *reflection in a vertical mirror*, *rotation by 180˚* and *glide reflection*. The collection of symmetries of a pattern is the *symmetry group* of the pattern. There are exactly seven distinct symmetry groups of frieze patterns.

1.3. Student Cognitive Processes. The five pedagogic principles laid down in [3],[4], and [5] guided the art module of the introductory liberal arts mathematics course, developed over a four-year period. During the first year, interviews were conducted on students who came during office hours for help; we assumed that the other students understood the material. The goal of the interviews was to clearly define problem areas as well as to identify teaching strategies that worked well.

To illustrate the issues, consider a pattern **P** consisting of a sequence of adjacent equilateral triangles alternately pointing up and down, and the modification **Q** of this pattern that adds dots to two vertices of each triangle, as shown below.

P  **Q** 

Q only possesses *translation* and *glide* symmetries while **P** possesses additional *vertical reflection* and *rotational* symmetries. We could summarize by saying that the motif of **Q** contains two *themes*—the triangle theme and the dot theme, while the motif of **P** contains just the triangle theme. Multiple-theme motifs increase pattern recognition error in the weaker students. However, student interviews revealed that coaching to identify the 'clock positions' of various pattern items, such as dots and vertices, provided students with a computational tool that they could use to verify the presence or absence of symmetries.

A problem also arose with respect to notation. Although flowcharts for naming the frieze patterns [2] are straightforward to use, non-technical students found use of these cumbersome. They could not, for example, identify several frieze patterns in a minute or two. Student interviews revealed that identification of patterns became quick when the students simply tested each frieze pattern for each of the five possible symmetries.

1.4. The problem domain. The interviews suggested defining the problem domain of clock-motifs and clock-patterns. A *clock-motif* consists of several rays emanating from a common center, possibly with varying lengths—for example, **Y, V, X, I**. These produce *clock patterns* by translation.

1.5. Algorithm. We created a computer tutorial that presented translations of randomly created clock-motifs on a screen and allowed the student to guess its symmetry group an unlimited number of times before proceeding to the next example [3].

1.6. Teaching strategy. We present four useful teaching strategies that arose from the student interviews and resulting model of student cognitive processes. (a) Student identification of patterns was greatly facilitated by identifying clock positions (in minutes) on the patterns. (b) The following computational theorem was provided to check on symmetries: A clock-motif with a ray at minute position p has violated *vertical reflection*, *rotational*, or *horizontal reflection* symmetry respectively, if a ray of equal length is not located at the following clock positions: $60 - p$ (mod 60), $30 + p$ (mod 60), or $30 - p$ (mod 60). (c) To facilitate dealing with multi-thematic patterns the tutorial produced clock-motifs with 4 to16 rays. (d) To facilitate quick naming of patterns, each tutorial screen flashed the five symmetries with familiar letters or symbols that illustrated the symmetries; for example, **v**ertical reflection (**V**), **h**orizontal reflection (<), etc. The name of the symmetry group of a pattern was a simple listing of the initials of the symmetries present, in alphabetic order. For example, the patterns **P** and **Q** presented earlier have symmetry groups **grtv** and **gv**, respectively.

1.7. Problem richness. Clock-patterns are not meant to replace the sophisticated examples from diverse cultures presented in [1] and [2], but rather complement these real-life examples. Student practice with *clock-patterns* improved their recognition of real-life examples. These observations are anecdotal; no formal statistical measure of improvement was carried out.

References

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