The Lost Harmonic Law of the Bible

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Abstract

The ethnomusicologist Ernest McClain has shown that metaphors based on the musical scale appear throughout the great sacred and philosophical works of the ancient world. This paper will present an introduction to McClain's harmonic system and how it sheds light on the Old Testament.

1. Introduction

Forty years ago the ethnomusicologist Ernest McClain began to study musical metaphors that appeared in the great sacred and philosophical works of the ancient world. These included the Rg Veda, the dialogues of Plato, and most recently, the Old and New Testaments. I have described his harmonic system and referred to many of his papers and books in my book, *Beyond Measure* (World Scientific; 2001). Apart from its value in providing new meaning to ancient texts, McClain's harmonic analysis provides valuable insight into musical theory and mathematics both ancient and modern.
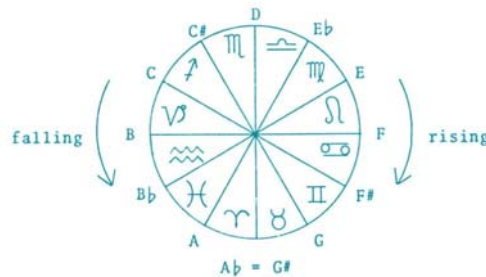
2. Musical Fundamentals

Figure 1. *Tone circle as a Single-wheeled Chariot of the Sun (Rg Veda)*
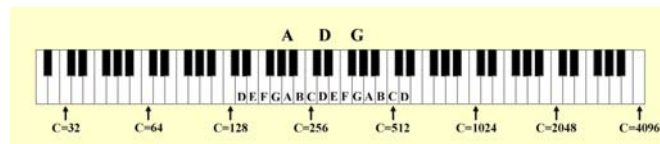

Figure 2. *The piano has 88 keys spanning seven octaves and twelve musical fifths.*

The chromatic musical scale has twelve tones, or semitone intervals, which may be pictured on the face of a clock or along the zodiac referred to in the Rg Veda as the "Single-wheeled Chariot of the Sun." shown in Fig. 1, with the fundamental tone placed atop the tone circle and associated in ancient sacred texts with "Deity." The tones are denoted by the first seven letters of the alphabet augmented and diminished by and sharps (♯) and flats (♭). For reasons that will become evident, we will choose D as the fundamental although the musical scale is indifferent to which tone is taken. If the fundamental is assigned a relative frequency of 1 unit then proceeding clockwise around the tone circle one arrives at the identical tone one octave higher with a relative frequency of 2. A revolution in the counterclockwise direction results in a

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tone one octave lower at a relative frequency of ½. It is the miracle of music that if the ratio of frequencies of two tones is a multiple of 2, the tones are perceived by the ear to be identical. As a result each tone will be considered to be a member of a pitch class of tones differing in frequency by a multiple of 2. The norm in the Old Testament is to use a double octave; i.e., 1:2::2:4, which also corresponds to Hebrews gematria for "Eden," 124, where the Hebrew letters were given numerical values.   The chromatic scale in Fig. 1 equally divides the tone circle and as a result is referred to as an equal-tempered scale. Each tone is given a value of 100 cents on a logarithmic scale with 1200 cents to the octave. At the bottom of the tone circle, at a relative frequency of $\sqrt{2}$ is the tritone, the most dissonant interval of the chromatic scale.

   Relative to the fundamental, the interval of the musical fifth (DEFGA) is found at 7 o'clock spanning 7 semitones. Its complement in the octave, 5 semitones, is called the musical fourth (DEFG) which arrives at the same pitch class as the falling musical fifth.   The interval of the major third has a relative frequency of $\sqrt[3]{2}$ and is found at 4 o'clock. Its complement is the minor 6th at 8 o'clock, the position of the falling major 3rd. The remaining interval of importance to Western music is the minor 3rd found at 3 o'clock and its complement at 9 o'clock, the major 6th. The structure of music is built around chords consisting of the fundamental, major 3rd, and 5th known as the major triad and the fundamental, minor 3rd and 5th , the minor triad. The unison (1:1 ratio), octave, 4th, 5th, major and minor 3rds, and major and minor 6ths are the only consonant intervals of the chromatic scale.

   The piano has 88 keys spanning seven octaves and 12 fifths as shown in Fig. 2 where, the piano, beginning with any tone, after 12 musical fifths one ends on the same tone seven octaves higher as shown in Fig. 3. Seven consecutive tones on the white keys gives rise to the heptatonic scale. A double octave is shown in Fig. 2 beginning on D; the double octave is the basic unit of the Bible. McClain refers to this mode as being the "menorah model" since it suggests the seven-branched candlestick referred to in Exodus 25:31-40 and found today in all synagogues. However, there are seven different modes of this scale depending on the choice of fundamental. The famous do re mi … scale begins on C.   This leaves the blacks positioned to sound the pentatonic patterns by themselves.

Figure 3. *Circle of fifths*

Figure 4. *Rising and falling fifths as twin tones.*

In the ancient world, the musical scale was defined by ratio of string lengths. Beginning with a length of string, placing the bridge at the midpoint of the string and plucking the remaining ½, one gets a tone one octave higher. Moving the bridge to a position where 2/3 of the string is plucked results in to a fifth, ¾ is a fourth.   These were the principle intervals of Greek harmonic law making up the tetrachord of Pythagoras immortalized by Raphael in his painting Birth of Athens. The Just scale also uses 4/5 a major 3rd, and 5/6 a minor 3rd. Increasing the length of the strings by the inverse of these ratios give rise to the same intervals but in a falling direction.   Therefore, 5/4 is a falling major 3rd and raising it an octave gives rise to 5/8, its complement the minor 6th. Likewise 6/5 is a falling minor 3rd while its complement is

the major 6th a ratio of 3/5. The inverse of the relative string length is the relative frequency. A summary of the consonant intervals of Western music is given as:

(interval, No. of semitones (s), ratio): (unison, 0s, 1/1), (minor 3rd, 3s, 6/5), (major 3rd, 4s, 5/4), (fourth, 5s, 4/3), (fifth, 7s, 3/2), (minor 6th, 8s, 8/5), (major 6th, 9s, 5/3), (octave, 12s, 2/1)

When the tones are defined by rational numbers the tone circle is no longer equally divided. . On a monochord, 12 successive fifths no longer results in a tone in the same pitch class but to a slightly altered tone differing by approximately one-quarter of a semitone (24 cents), called the Pythagorean comma. The Pythagorean comma is a rich source of metaphor in ancient texts.

3. Twin tones

Consider Deity at D and the twin tones A and G shown in Fig. 4 serving Plato as harmonic and arithmetic means in the octave and achieved by rising and falling fifths. In Bible mythology, these symmetrical tones represent either twins or archangels found in the Bible. We will associate it in Sec.8 with Isaac's twin sons, Jacob and Esau. If D is given the value of 1 then A and G, being rising and falling musical fifths are 3/2 and 2/3 respectively, where we are taking the ratios to mean relative frequency (the inverse of string length), and D is the geometric mean of A and G. Notice that the octave interval subdivides into two musical fourths (5 semitones each) and a wholetone (two semitones). This also demonstrates the most fundamental principal of harmonic law, that intervals add while frequency ratios multiply, i.e., 5+2+5= 12 while 4/3 x 9/8 x 4/3 = 2 also 4/3 x 9/8 = 3/2 or 5 + 2 = 7.

The integer 2 was considered to be the female number which can give birth to no new tones without the participation of the male number 3. Since tones differing by a multiple of 2 are in the same pitch class and are considered to be identical, multiples of 2 are suppressed and this triple of tones is represented by, 3^{-1} , 1, 3 as shown in Fig. 4. The second row of numbers in Fig. 4 is derived by multiplying the first row by the common denominator 3. Since A is the largest integer at 9, the fundamental D is multiplied by the smallest power of 2 to enclose 9. In other words an octave 12/6 is created between D and D' an octave higher that encloses 9. The tones G and D must be multiplied by powers of 2 to "seal" them in the 12/6 octave. The integers 8 and 9 are the harmonic and arithmetic means, respectively, of 6 and 12. In Epinomis (991) Plato states that "in the potency of the mean between these terms (6,12) with its double sense, we have a gift from the blessed choir of Muses to which mankind owes the boon of the play of consonance and measure, with all they contribute to rhythm and melody."

Opposite Deity is the dreaded tritone, "diabolus in musica" and represents the worst possible dissonance, but more important, it betrays a fundamental asymmetry in the middle of the tone circle as we shall see in the Sec.5. Its relative frequency is represented by $\sqrt{2}$ on the equal-tempered scale. Without the concept of irrational number, ancient cultures found ways to accurately approximate $\sqrt{2}$ by rational numbers. It is from the portion of the tone circle surrounding the tritone that, in metaphoric terms, the savior will be born.

Fig. 4 has been hypothesized by McClain to represent the heroic Jewish figure, David, since David's name in Hebrew gematria is 464 where 4:6 and 6:4 are increasing and decreasing fifths. McClain suggests that David's sling which he so deftly used to kill Goliath is inverted in Fig. 4.

4. Pentatonic scale

The pentatonic scale shown in Fig 5 is generated from its center at D by two successive rising and falling musical fifths as follows:
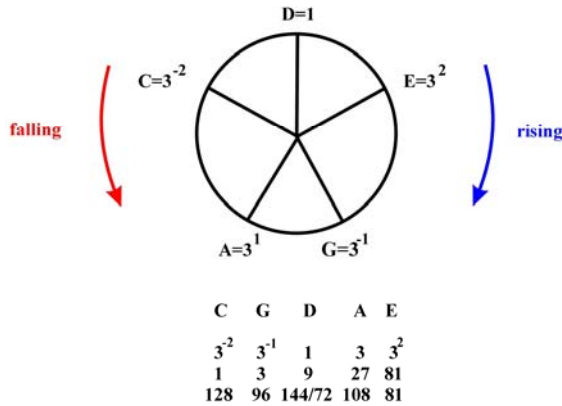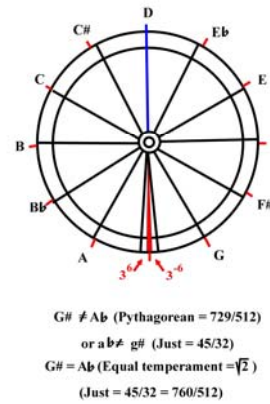
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Figure 5. *Pentatonic scale* **Figure 6.** *The Pythagorean comma*

Again, the first row of numbers is multiplied by the common denominator 9 to get the second row after which the terms are multiplied by powers of 2 to place them in the 144/72 octave which seals the set symmetrically with DEGACD as the scale order. Notice that the tones of the pentatonic scale occupy symmetric positions around the tone circle. Symmetric scales result from the choice of D as fundamental. Semitone intervals are dissonant or harsh to the ear. Since there are no semitone intervals in the pentatonic scale, any combination of tones has a pleasing sound. The pentatonic double octave is: 72:144::144:288 where 288 represents David's " 288 trained singers, sons of Asaph, Jeduthun, and Heman" (1 Chron. 25:1-8).

A similar analysis can be made for the heptatonic scale with three rising and falling fifths.

5. Pythagorean Comma

Six rising and falling fifths result in two representations of the tritone leaving a small gap between them, the Pythagorean comma, enclosing the equal-tempered value at $\sqrt{2}$ as shown in Fig. 6. With D as the fundamental, these rising and falling fifths are symmetrically placed around the tone circle and make up 11 of the 12 tones of the "Pythagorean scale." The last pair produces an asymmetric 13th tone in the middle of the tone circle which is the source of many metaphors in the Bible and other ancient texts. If D is the fundamental, then the pair of tritone approximations are G $\sharp$ and A_b, two tones that are identical on the piano. These three tones are represented by integers as:

$$\begin{array}{cccc} \text{Figure 6a} & \text{Gsharp} & \text{D} & \text{Aflat} \\ & 3^{-6} & 1 & 3^{6} \\ & 1 & 729 & 531441 = 3^{12} \end{array}$$

If "1" in the second row is multiplied by $2^{19} = 524288$ with a tail of David's 288 "trained singers", the ratio 531441: 524288 is the Pythagorean comma and represents the amount by which twelve musical fifths deviate from seven octaves ($2^7 \approx (\frac{3}{2})^{12}$ or $2^{19} \approx 3^{12}$). Clearly, no power of 2 can equal a power of 3. The savior will be born in this small gap between G $\sharp$ and A_b. This gives rise to a wealth of sexual imagery in the Rg Veda in which this region is pictured as a "vagina" while the wedge to the center of the circle is a sepah or "penis." The whole construction arises from an interplay of "male" and "female" in which the commas that arise are a kind of "genital friction."

The savior is born in this womb. In fact, notice that G sharp at "1" can be made to represent Deity by multiplying it by 512 (nine powers of 2) and then switching places with 729 at D so that 512 is the fundamental and 729 now represents the tritone as shown in Fig. 6a. But $512 = 8^3$ while $729 = 3^6$ where

729/512 = 1.4238, slightly greater than $\sqrt{2}$. Is it too much to suggest, as McClain does, that $8^3 \equiv 888$, the number assigned the savior, in Biblical times Isaac and in later times Jesus Christ, while $3^6 \equiv 666$ is the "devil's number?" Pure doublings point to a misunderstood metaphor for "Israel" as YHWH's wife. Since the male number 3 is responsible for the small overlap between the first and thirteenth tones, McClain hypothesizes that this is the origin of the law requiring circumcision of males.

The twelfth tone of the chromatic scale is represented by the six digit number 531441. Clearly the ear can only perceive the first two digits. McClain feels that the "1" in the tail of the number represents the "forgotten cornerstone" while 1440 is the gematria for Adam. The tone circle from D to D' is made up of approximately 53 Pythagorean commas.

6. The Hebrew octave

Old Testament arithmetic is based on the numbers 1,2,3,4,5,6,7. In Genesis the first six days are used for the work of constructing the world while the seventh day is reserved for God. To construct tunings using musical fifths, only the numbers 1,2,3,4 and 6 were used, but we saw in the last section that the chromatic scale requires six digit numbers. There are references in the Bible to the "Anakim" or "giants" from whom the Chosen win the "Holy Land"(Deut. 1:28) with another tuning represented by smaller numbers. These Anakim correspond to the huge numbers and slightly excessive spiral fifths. In Greek mythology, 5 is the number assigned to humans; humans are "fivers." This number will reduce the numerosity of the chromatic scale from six digits to a mere two or three digits.

In the Bible, God's number seven is introduced since the Jews are God's people. The Bible octave for the "gestatation of the Savior" (Rev. 11-12) is:

$$35:49::50:70$$

where $49/35 = 70/50 = 7/5 \approx \sqrt{2}$, the simplest approximation of the square root of 2 using small integers. Therefore 49 and 50 enclose the tritone and all approximations to $\sqrt{2}$ occur within this tolerance on either side of the tritone. For example, the Pythagorean comma is included within this interval. But "seven Sabbaths of years" comprising 49 years constitutes the Jewish calendar cycle while the 50th year is the "Jubilee year" in which the land lies fallow (Lev. 25). The distance between 49 and 50 on the tone circle (34 cents) is 1% on either side of the tritone prompting McClain to associate this with New Testament references to a "tolerant God" in Matthew 18:12-14 and repeated in Luke 15:3-6. "Suppose a man has 100 sheep. If one of them strays, does he not leave the other 99 on the hillside and go in search of the one strayed." All approximations to the tritone occur within the 49-50 tolerance. This small errancy permits alternative metaphoric interpretations. It also points to God as a "craftsman god" as opposed to a god "scientific precision."

The number 70 is one of the most frequent numbers in the bible and generally reserved for God. At three places in the Bible 70 men are slain. For example, by the order of God, Ahab's 70 sons were killed, and their heads placed in baskets (Kings 2 10:1-7. In fact the Bible is the story of how the many gods of ancient civilizations were replaced by the "One God" and how a people, the Jews, were created to worship that God.

7. The Matrix

McClain's story of musical metaphor continues with the introduction of a matrix of integers based on multiples of the numbers 3 and 5. The bottom row of the matrix are powers of 3 while the rising edge are powers of 5. Elements within the matrix are multiples of both 3 and 5 as shown in Fig.7 beginning with 1 in the lower left-hand corner of the matrix and referred to by McClain as the "cornerstone" which will become the savior of the system.

The first twelve numbers of this matrix are: 1,3,5,9,15,25,27,45,75,81,125,135 which McClain assigns to The twelve sons of Jacob. Notice that the first four: 1,3,5,9, being single digit
are the sons of the "weak-eyed Leah," Jacob's first wife, while 125 is

Joseph's number and 135 belongs to Benjamin, the youngest son. The numbers 225 and 625 are assigned to Manasseh and Ephraim, the sons of Joseph. There are no more than twelve sons because Jacob dies at age 147.

$$
\begin{array}{ccccc}
 & 125 & & & \\
25 & 75 & 225 & 625 & \\
5 & 15 & 45 & 135 & \\
1 & 3 & 9 & 27 & 81
\end{array}
$$

Figure 7. *The matrix representing Jacob's family*

The 3,4,5- or 4,5,6-nature of the musical scale is revealed by the first triple of numbers 1,3,5 which is illustrated by

$$
\begin{array}{cccc}
5 & & 5 & \\
4 \quad 3 & & 4 \quad 6
\end{array}
$$

where the tone representing 1 has been inflated to 4 in the first triangle and the 3 is inflated to 6 in the second. Plato states in the Republic that "4,3 mated with 5, thrice increased, provides two harmonies (Republic 546a-d)" All the tones of the scale can be derived by a system of vectors from this 3,4,5-relation as shown in [4]. Consider the 4,5,6-triangle. It defines three intervals: 3:2, 5:4, and 6:5, a rising fifth, a rising major third, and rising minor third major third, the intervals that make up the major and minor triads of Western music.

Notice in Fig. 7 that each integer (frequency) in the matrix is the geometric mean of any pair of symmetrically placed integers, e.g., 15 is the geometric mean of 9 and 25, 3 and 75, and 1 and 225. As a result, the pairs may be thought of as inverses with respect to the central tone, e.g, 25/15 = 5/3 while 9/15 = 3/5. In this sense the matrix is able to define inverses in terms of integers.

8. The First Eight Sons of Jacob

Consider the first eight family members: 1,3,5,9,15,25,27,45. The next number in this sequence is 75, the age of Abram when he left Haran for the Holy Land (Gen. 12:5). Since every tribe presents to the tabernacle with "one silver plate whose worth was a hundred and thirty shekels" (Numb. 7), McClain suggests that Abram's tent was also worth 130 shekels, the sum of the first eight family members, each < < 60 in value.
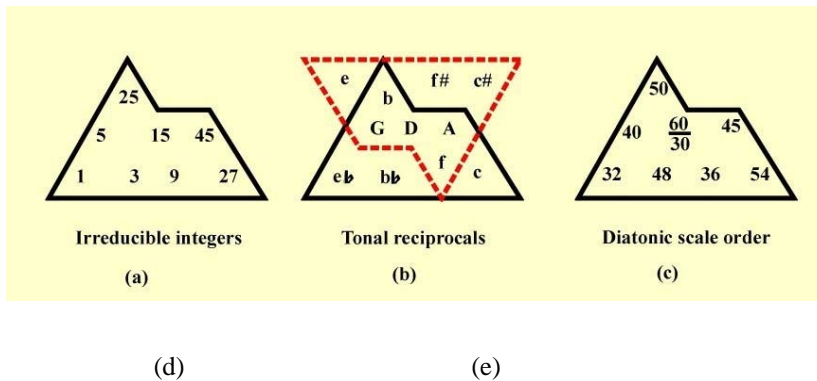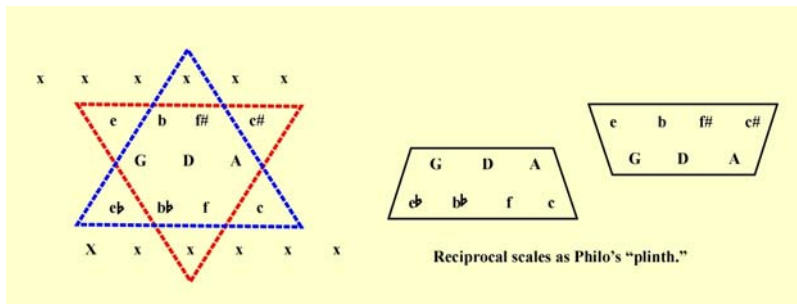

Figure 8. *The first eight members of Jacob's family create the "Davidic Tuning." The matrix in (a) is multiplied by powers of 2 into the 60/30 octave in (c) resulting in the tones in (b). The tones of (b) divide into a pair of plinths in (d) representing upward and down ward heptatonic scales. The tones fit within the Star of David.*

(d) (e)

These eight matrix members are shown in Fig. 8a in a tent-like configuration suggestive to McClain of Abram's tent. In Fig.8c the numbers are multiplied by powers of 2 to place them in the

60/30 octave sealing 45, a computation shared in ancient Sumeria by base 60 sexagesimal arithmetic. Deity at D in the midst of the twin tones A and G are found along the central axis. In this matrix we see that, with respect to Deity, the same relationships hold as for the 4,5,6-triangle above. For example, 45:30 = 3:2 while 40:60 = 2:3. Likewise 50:30 = 5:3 while 36:60 = 3:5. Since (36, 50) and (40,45) are the only two symmetric pairs with respect to Deity in this matrix they have special importance. Also note that 50 is the arithmetic mean of 40 and 60 that brace it from below forming the major triad G, b, D, while 48 is the harmonic mean of 40 and 60 that brace it from above, forming the minor triad G, b flat, D. Similar relationships abound throughout the matrix.

The Hebrew double octave is: 30:60::60:120 where "the sons of Asaph, Jeduthun, and Heman—with cymbals, harps and lyres, stood east of the altar with 120 priests who were trumpeters. (2Chron. 5:12)" The age of 120 is the natural lifespan of humans set out in the Bible (Gen. 6:3) and was the lifetime of Moses. In Sumerian mythology, 60 is the god number of Anu, sky god and father of the Pantheon, 40 is Enki, the god of sweet water, and 50 is Enlil, the mountain god and active ruler. It is well known that the Sumerian Gilgamesh legend has many similarities to the story of Genesis transformed by the authors of the Old Testament to accommodate Jewish meanings.

In Fig. 8c, the relative frequencies of Fig. 8b are related to tones from what McClain refers to as the Davidic tuning for reasons that will soon become clear. Upper case letters are reserved for the tones defined using only the integers 2 and 3 as before. Lower case letters are the tones created by the integer 5. Tones from the circle of fifths lie on the central axis of the matrix and are often referred to as the "Pythagorean scale." The tones from the central axis and the two adjacent rows make up the "Just scale." Major triads of Western music appear in every upright triangle of nearest neighbors (above or below), and vice versa (e.g., GbD). Triangles of tones from the central axis and the row below results in minor triads (e.g., Gb$_b$D). There is a reference in the Gilgamesh legend to the central axis with its two neighboring rows as a "three-ply rope that is hard to break."

You will notice that McClain has introduced a pair of matrices that he refers to by the Hindu expression, "yantra." The upward yantra is the one in Figs. 8a and 8c while the downward yantra in Fig. 8b is turned at 180 deg. and its tones are "inverse" of the frequencies in Fig. 8c. All tones in the intersection of upward and downward yantras have inverses that can be expressed in terms of integers, i.e., each integer has a symmetric twin within the intersection. Although McClain has never found direct evidence for his yantras, the Hebrews' journey from Sinai to Edom with YHWH in the midst of "a pillar of cloud by day and in a pillar of fire at night (Numb. 14)" Could this refer to the downward yantra and upward yantras?

Fig. 8b is divided into a pair of upward and downward pointed plinths in Fig. 8d. These plinths comprise the rising and falling ancient Dorian scale with D as the fundamental having all the intervals of the do re mi…scale but in reverse order. The rising scales fit within the 60/30 octave and is represented as Davidic tunings since the tones fit easily into the star David as shown in Fig. 8e. As double octaves these tunings extend to 120 and 288 (the "120 trumpeters and 288 trained singers").

Figure 9. *The upward and downward heptatonic Dorian modes lie in the 60/30 and 144/72 octaves respectively. The double octaves have limits of 120 and 288, "David's singers" and the "priests who played trumpets."*

The mode of the Pentateuch and its symmetric opposite.

D	e♭	f	G	A	b♭	c	D
D	c♯	b	A	G	f♯	e	D
30/60	32/64	36/72	40/80	45/90	48/96	54/108	60/120
144/288	135/270	120/240	108/216	96/192	90/180	80/160	72/144
	16/15	9/8	10/9	9/8	16/15	9/8	10/9
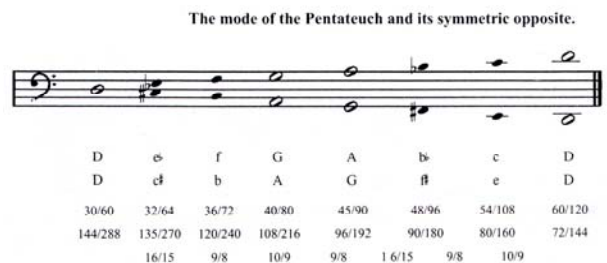
Along with the self-symmetric Phrygian mode, the Dorian scale comprises the two revered modes of ancient music.

McClain's matrices are very versatile and are used and reused to tell many stories. For example, consider 40 and 45 straddling Deity at 60/30. As mentioned above, 40 and 45 are the harmonic and arithmetic means of 30 and 60, respectively. They are also the twin tones mentioned in Sec. 3. Rebecca gave birth to the twins, Jacob and Esau when Isaac was 60 years old. Since Jacob was the younger, he

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gets the larger number 45 while Esau is 40. But according to the famous Bible story, Jacob stole Esau's birthright. This identity theft was necessary for Jacob, renamed Israel, and become the father of the Hebrews. We shall see in the next section that the number 45 is needed for this purpose. The number 15 from the matrix inflates to the Sumerian god number 60, symbolic of the "old" system. Abram at age 75 emerges from this yantra whose numbers sum to 130 as if the "triangle" and its shoulder (see Fig. 8b) were the family tent (McClain's metaphor). The number 15 will be replaced by the next number of the matrix 45 symbolic of the "new" system and representative of the patriarchs Isaac and Jacob. Abram will also make another appearance in this new yantra.

9. The Patriarch's Matrix

Consider Jacob's family matrix shown in Fig. 10. Using the same symmetry argument, there are 5 pairs of integers in symmetry around Deity at 45 (Jacob's number): (15,135), (27,75), (9,225), (3,625), and (25,81), the five pairs in inverse symmetry with respect to 45 in the family matrix of Fig. 6.

Figure 10. *The calendar matrix or "matrix of the Patriarchs" seals 11 of the 12 tones of the chromatic scale in the 720/360 octave with inverse symmetry. A hexagon of tones fit into "star of David."*

Jacob's number 45 is transformed as:
$45 \rightarrow 90 \rightarrow 180 \rightarrow 360 \rightarrow 720$ into the 720/360 octave which is the least common multiple of 72 and 60, the octave limits of the Davidic tunings of Fig. 9. Sarah was 90 when Isaac was born while Isaac died at 180. All twelve tones of Jacob's family matrix are sealed by the 180/90 octave. Within the 720/360 octave, eleven of the twelve tones are sealed with inverse symmetry as shown in Fig. 10b in the intersection of the upward and downward yantras and referred to b**y** Plato in "Laws" as Poseiden's five twin sons. They are the eleven tones of the Just scale that can be expressed as the ratio of small integers, with the remaining tritone expressible as an awkward approximation to $\sqrt{2}$ under a "tolerant" Deity accustomed to such approximations. The base 60 system has now been transformed to a base 10 system inspiring the Hebrew importance of ten-ness. Notice that the pentatonic scale: CGDAE appears on the central axis defined again from Deity at the center. Tone pairs, such as c,C and e,E differ from each other by the ratio 81:80 (approximately 22 cents), known as the syntonic comma.
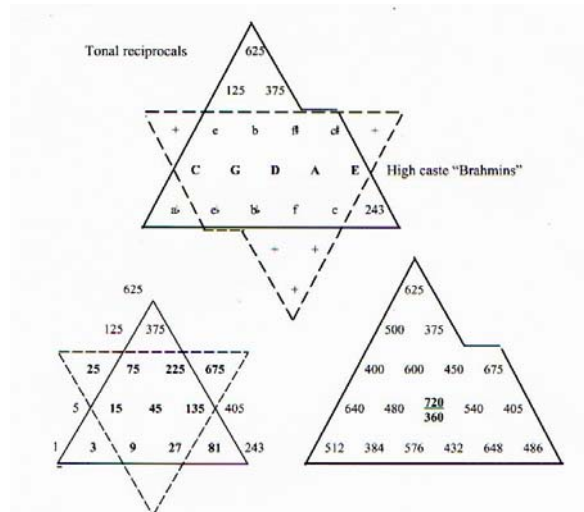
The hexagon in the midst of Fig 10b illustrates the rotational symmetry of the tone circle. The nor 3rd at f rotates to its complement, the major 6th at b; the major 3rd at f sharp rotates to its complement, the minor 6th at b flat; the fifth at A rotates to it complement, the 4th at G. Also the major triad (D, f ♯, A) of the rising scale maps to the major triad of the falling scale (D, b$_b$, G) while the minor triad of the rising scale (D, f, A) maps to the minor triad of the falling scale (D, b, G). Notice that hexagon as shown also in Fig. 8d and 10c with its Magen David triangles eliminates all elements lacking symmetric opposites. We see that the "glory of the Shekinah" in Fig 10c excludes Plato's syntonic commas at 400:405 and 640:648 to reveal the supreme elegance of YHWH's reductionism.

This matrix is considered to be the matrix of the "patriarchs" or the "calendar" matrix. In ancient cultures, 360 was considered to be the "canonical year" even though these societies were quite aware that this was not the length of either the solar or lunar years. In fact the ratio of 360

days to the lunar year of 354 days and the solar year of 365.25 days is remarkably close to the Pythagorean comma. The thirteen tones with inverse symmetry within the Patriarch's matrix and its inverse occupy symmetric positions around the tone circle shown in Fig. 11. Notice that the cornerstone at 512 makes a good candidate for the twelfth tone, G sharp, approximating the tritone at $\sqrt{2} \approx 720/512 = 45/32 = 1.406\ldots$ In a sense it "saves" the octave. In Sec. 5 we witnessed its birth in the region around the tritone .

10. The Marduk Matrix

Note in Sec. 8 and 9 that the six figure numerosity of the chromatic scale (see Sec. 5) has been reduced to a limit of 60/30 or 720/360. In Fig. 12 McClain introduces a matrix, the Marduk/Elohim matrix, with the ferocious numerosity, 8,640,000,000/4,320,000,000 which he attributes to Marduk (c.2000 BC) the head of the Babylonian Pantheon, whose only invariances under rotation are the "seven-headed dragon" as menorah scale, and it contains the Patriarch's yantra surrounding Deity. It is 15 rows high in perfect inverse symmetry on its
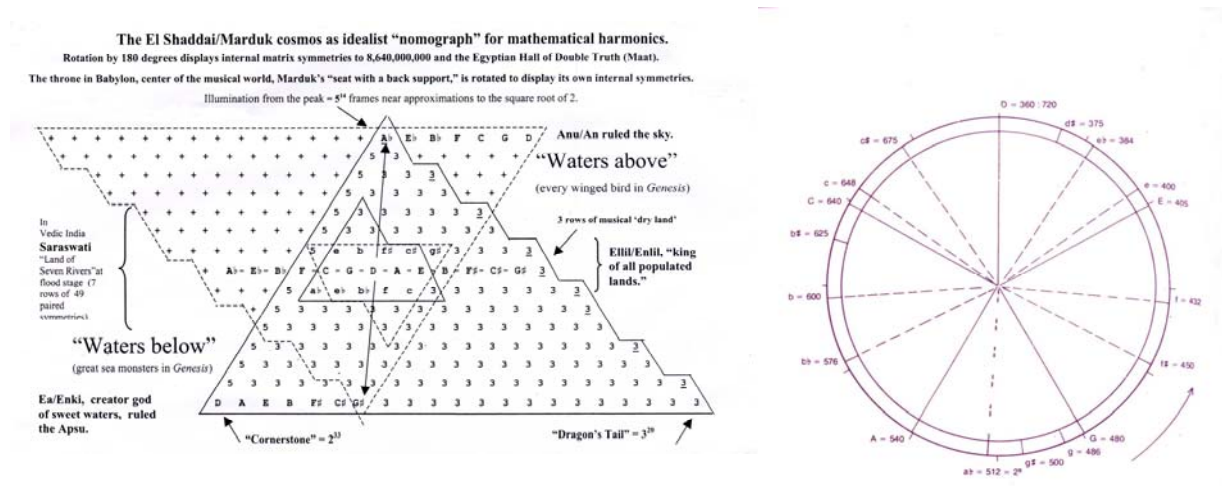

Figure 11. The calendar matrix defines six pairs of tones with inverse symmetry with respect to the fundamental D.
Figure 12. The Marduk/Elohim matrix.

central axis. The ratio of the highest and lowest tones, A flat and G sharp, to the fundamental give three decimal point approximations to $\sqrt{2}$ with the gap between A flat and G sharp now a mere 4 cents. McClain associates this matrix with the Biblical "flood" which rose 15 cubits in depth (Gen. 7:21). The Patriarch's matrix has been lifted seven rows in the air. Seven rows of this matrix have exactly seven counters, sharing reciprocity, reflecting the "seven sabbaths of years, that is seven times seven years" described in Sec 6. Notice the rows of 21 tones along the bottom of the matrix and 21 tones facing them along the top row of the inverted yantra. In Babylonian mythology, the dragon of the deep, Tiamat, has 20 dragon offspring. But the matrix is also important in Hindu mythology where one thousand Maha Yugas consitute a Kalpa of 4,320,000,000 "years;"4,320,000,000 and its double is Brahma - the Immense being - a cycle of 8,640,000,000 years. A Hebrew myth describes the digging of the Temple foundation to a depth of fifteen "cubits" and fifteen psalms are labeled "a song of ascents." In Egyptian mythology, fourteen steps lead upwards to the throne of Osiris—while forty-two judges look on from either side in the Hall of Double Truth: occupying 21 seats along opposite rows as in our matrix.

11. The YHWH Matrix

The enormous numerosity of the Marduk matrix can be reduced to exactly 9% of its value while at the same time accommodating more tones with inverse symmetry including the eleven tones of the octave in perfect inverse symmetry along the central axis shown in Fig. 13.
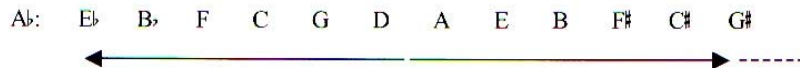

Figure 13. *The central axis of the YHWH matrix.*
Five tones are in inverse symmetry about the fundamental D (between the arrows). Twelve tones are sealed by 777,600,000 (outer brackets). The 1st and 13th tones, A flat and G sharp differ by the Pythagorean comma.

Consider the gematria value of YHWH which is: 10.5.6.5. McClain interprets this as 10^5 x 6^5 = 777,600,000 and assigned to D (Deity); God's number 7 is repeated thrice while 600,000 are the number of bricklayers who fled Egypt with Moses. Fig. 13 shows God surrounded by his minion of ten men along the central axis in perfect inverse symmetry (inner brackets) with 12 tones sealed in the octave (outer brackets) and all thirteen tones along the axis. The three ply rope now extends to the entire circle of fifths and now includes major and minor triads in each of the twelve keys. When placed within a single octave the twelve tones form a "serpent tuning" in which the 1st and 13th tones differ only by the Pythagorean comma suggestive of the "ouribus" figure in which a snake eats its own tail. According to Isaiah 27: "On that day the Lord with his cruel and great and strong sword will punish Leviathan the fleeing serpent, Leviathan the twisting serpent and he will kill the dragon that is in the sea." McClain feels that all serpent myths depict this "serpent tuning" shown in Fig. 14, and that the 21 elements in the figure are an extension of the foundation in Fig. 4 ad account for the 21 elements in the base of the Marduk matrix in Fig. 11 and inspire the 42 gods of the Egyptians.
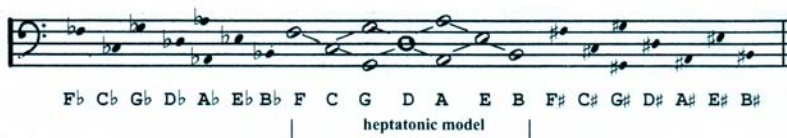.

Figure 14. *The serpent tuning. All tones of the chromatic scale are places in scale order. A flat and G sharp differ by the Pythagorean comma giving rise to the metaphor of the "snake eating its own tail."*

Finally, the number 243, the last number in the bottom row of Fig. 10a, is the gematria sum of the letters of Abram, the father of the Jewish people before his name is changed to Abraham. But 243 x 32= 7776, the lead digits of YHWH. In Numbers 31, 32,000 virgins, with its lead digits 32, are among the spoils in the genocidal victory of the Israelites over the Midianite Kingdom. We see here Abraham lifted to the throne of God in order to identify God with his "people." A bloody and senseless episode of the Bible is shown in its true colors to be a brilliant piece of story-telling in the service of creating God's people.

12. Conclusion

This paper is meant as an introduction to Ernest McClain's Bible studies. McClain has produced impressive evidence to show that a lost knowledge of harmonics was used to produce numerous metaphors in the Bible with each metaphor having multiple meanings. An understanding of these hidden connections gives greater meaning to this great book while placing it squarely in the context of other ancient civilizations that also drew upon these ideas.