D-Forms: 3D forms from two 2D sheets

Tony Wills

Wills Watson + Associates

10 Greenham Road

London N10 1LP UK

W: www.wills-watson.co.uk

E: tony@wills-watson.co.uk

Abstract

Is there a significant branch of geometry that has been overlooked? Unlikely as it may seem, D-Form geometry provides designers, architects, sculptors and artists with a vast, new vocabulary of three-dimensional forms that are easy to play with and make. Easy as they are to fabricate, D-Forms are proving equally hard to predict with computing. This geometry exploits some interesting properties of developable surfaces that, among other things, will enable you to 'square the circle'.

Introduction

D-Forms are similar to forms used by sculptors such as Constantin Brancusi, Barbara Hepworth, Alexander Calder and Isamu Noguchi. At the same time, D-Forms are created with the seemingly simple mathematics of developable surfaces.

Peculiar as it may seem, I had the initial idea for D-Form geometry in a dream. Fortunately I had a pen and paper by my bedside and jotted down the details. Given that the structure of the benzene ring appeared to Friedrich August Von Kekule in a dream, then this might be good company to be in and not so strange after all.

We all have a vocabulary of forms in our heads and, as a product designer, I constantly update and refer to mine in the course of my work. So, the idea that there exists a large (possibly infinitely large) family of simple forms out there that I had never before encountered was something of a surprise to me – as I hope it will be to you. I have noticed that some of the people with whom I have shared this concept have developed a kind of D-Form obsession so maybe this paper should come with a mental health warning.

What is a D-Form?

Firstly, a D-Form is a kind of three dimensional equation: joining the edges of flat surface S1 plus flat surface S2 (which must have identical perimeter lengths) at initial points of contact A and B1 forms (equals) a specific 3D shape. For example, referring to figure 1a, by taking a pair of ellipses and joining point A on one to B1, B2, B3, B4 and B5 you will produce a series of 3D shapes that progressively change their form from one to the other. Depending on the ratio of the minor and major axes together and where the A and B points are placed, a whole range of D-Forms will appear. Five variations are shown in figure 1b.
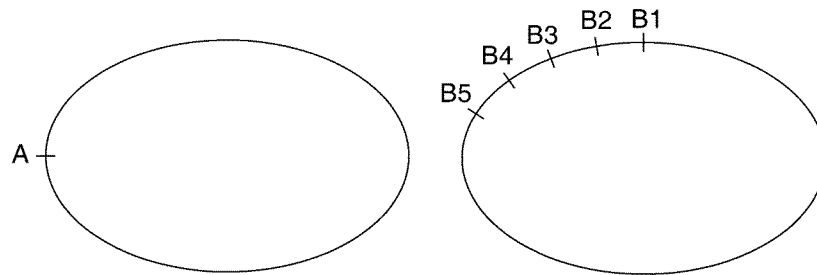
Figure 1a, two identical ellipses. Start joining A to B5, or B4, or B3 or B2 or B1 and different D-Form 'solids' will result as in Figure 1b
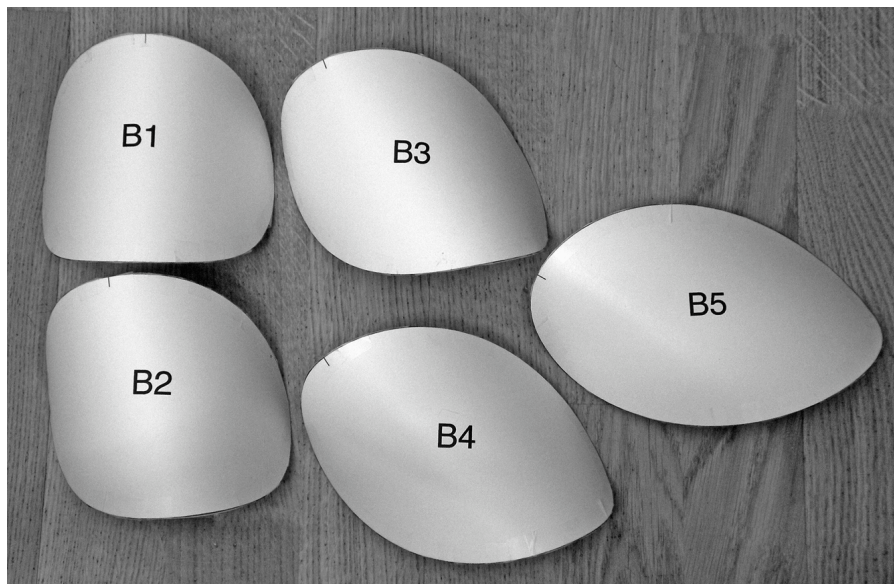

Figure 1b

Making a basic D-Forms

The first D-Form I stumbled across was simply this combination of two identical ellipses. When the initial joining points are the ends of the axes (major on one ellipse joined to the minor on the other) the result has a D-shaped cross section. To make a D-Form like this see figure 1a and join points A and B1. Draw the ellipses on a computer or construct them in the normal way. You can even draw a freehand shape like an ellipse. Cut them out from stiff paper or thin card. **Tip:** when identical shapes are being used, draw one and cut out two sheets at once thus ensuring identical shapes and therefore perimeter lengths. Choose a point A on one of the cut outs and a point B on the other but make sure A and B are in different places otherwise the form will lack and three dimension shape. Start taping the two perimeters of the cut outs together using short lengths of tape. **Tip:** I use a heavy tape dispenser and 'invisible' tape making it easy to tear off short lengths with one hand whilst the other manipulates the emerging D-Form. After a few minutes of taping you will see that each cut out is 'telling' the other what to do. It is as if this D-Form is a kind of three-dimensional equation where the sum of the two cut outs being joined, starting from the selected A and B points, equals a specific D-Form.

Materials

For practical purposes, sheets of relatively inelastic material like card, thin plywood, steel or aluminium are most likely to achieve a satisfactory result. Material stiffness and its relative thickness have a direct result on the scale of a D-Form that can be made and the scale is also affected by the tightness of curves required in the final form. This is a matter of experience and I have built D-Forms from a few centimetres across to over 4 metres high in many different materials. Mistakes I made include laminate that was too floppy and collapsed under its own weight to 8mm plywood that took ten men to force into shape. Steel sheet can be used and I have had quite large sheets cut with a laser so that tabs are added to the edges enabling them to be folded over during construction much like an old fashion tin toy. I have tried photo-etching thin sheets of brass and this works very well. A silversmith has made silver soldered D-Forms in bronze sheet for me but this can be expensive. Mesh or woven surfaces that can distort from rectangles to parallelograms are not 'D-Formable' although some interesting forms might still result as some fabric designers have found. Transparent drafting tape in a dispenser is best for joining card and I have successfully used a glue gun to internally join aero-ply D-Forms. If polypropylene is used as a skin on truncated D-Forms, filled resin can then be poured into the cavity and a cast made. This last method is particularly effective for maquettes of larger sculptures.

Variations

Some D-Forms where one or both of the sheets have vertices, will require creasing so they appear to have more than one developable surface. The 'Squaricle' (figure 4) is an example of this. The templates for making it are shown in figures 3.
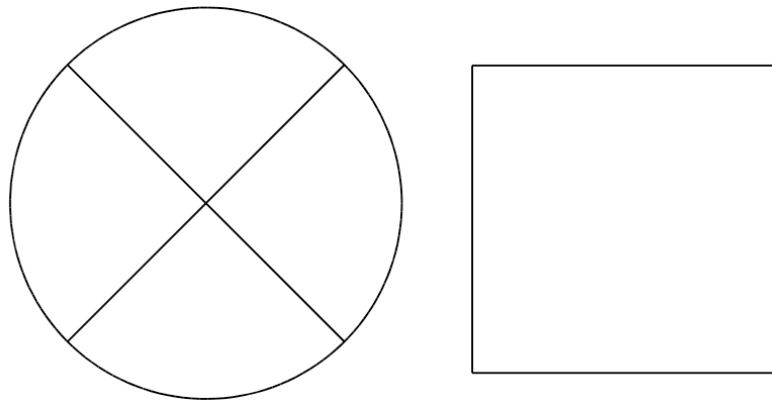

Figure 3, Squaricle templates Both lines on the circle are creased as 'crests' and the ends of the creases are joined to the corners of the square.

An alternative solution discovered by Prof. Tom Banchoff Brown University requires that the creases are made on a square drawn within the circle touching its perimeter. This gives a much flatter form. Other solutions are no doubt possible. Ellipses and rectangles are obvious contenders for variations on this theme.
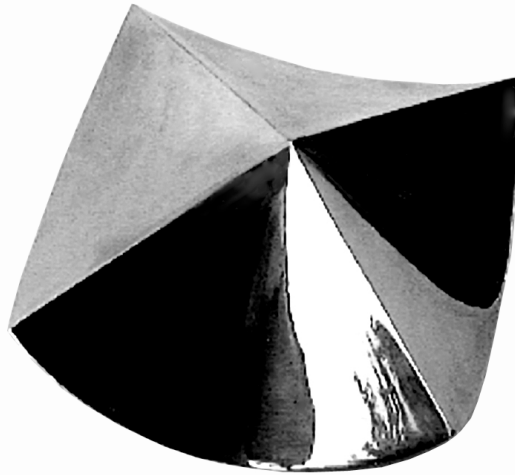
Figure 4, Squaricle made from gold plated bronze

Existing D-Forms

There are some existing D-Forms in the world. For example, I recently saw a French street light with a D-Form luminaire made from a circle and an ellipse. Leather workers sometimes use basic D-Form geometry to make bags and upholsters make forms from just two surfaces of fabric but these tend to stretch depending on the warp and weft. In general, I have found that these existing shapes tend to be symmetrical about at least one axis. Taken to its extreme, D-Forming could be said to include a cube since by joining two rectangles (ratio 1:3 with creases a third the way along) then the cube can be made with one joined edge. However, it is the uncharted territory, particularly of twisted D-Forms, that interests me. The D-Form family is clearly very large and is a little like discovering a new continent, some coastal parts are already inhabited but the hinterland is virgin territory.

One might think that nature has used D-Forms, for example as seed-pods, but this does not appear to be the case. The closest natural phenomenon might be the structures made by weaver ants or tailor-birds where two leaves are sewn together to form nests.

Extending the concept

As a rule, the more simple the concept the more potential it contains. D-Forming is a very simple idea and, with the input of my colleagues, is living up to this observation. In my experience, ideas arise and are subsequently developed out of a context that usually includes respected collaborators who ask well-defined questions. The D-Form concept has been developed in this way. I took the first D-Form to one of my engineering colleagues Dr Philip Davies, who challenged me to try combining not two identical shapes but two entirely different ones. To our surprise, although some were more difficult to form than others, all the shapes and combinations we tried produced elegant D-Forms. One of these early tests was the Squaricle, created from the combination of a square and a circle, see figures 3 and 4. Whilst I don't claim to have 'squared the circle', it is an interesting form with several possible outcomes depending on how the circle is creased. Many other D-Forms followed from this simple experiment.
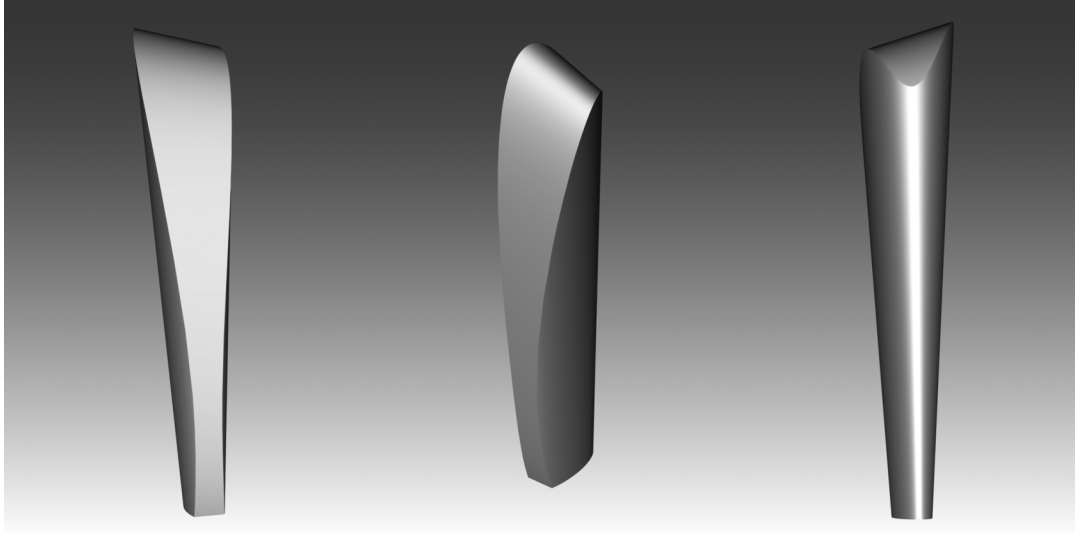
Figure 5, example of a truncated D-Form seen from various angles. This is a sketch for a stainless steel sculpture.

Anti D-Forms

Anti D-Forms came out of my collaboration with John Sharp. In one of our working sessions, rather than work with pairs of surfaces, we decided to try to join two holes with equal perimeters. Not only did this work but we found we could take the surfaces that we had removed to make the holes, construct the 'positive' D-Form from them and insert it precisely within the anti D-Form. After this, we tried half positive and half negative D-Forms. You can try this by cutting an ellipse out of a sheet and reintroducing it at an angle. We have recently asked what would happen if we didn't cut out the shapes at all, just joined two lines of equal length that are drawn on two surfaces. At the time of writing results are inconclusive but looks as if some D-Forms are possible using this method. Figure 6 shows an anti-D-Form.

Figure 6, a pair of ellipses with elliptical holes that are joined together making an anti-D-Form

507

A general rule?

If D-Forms only worked with ellipses then a 'special rule' would be at work here. I am not a mathematician so I don't know how to about proving that a 'general rule' applies. However, it appears that there is at least a very large number of pairs of surfaces (with the same perimeter length) that, in combination and with different starting points, can create an even larger number of unique D-Forms. I have shown the D-Forms to amongst others, Prof Helmut Pottmann at the Institut Fuer Geometrie, Vienna. He describes D-Forms from a geometer's view point as an 'open problem' and describes them like this:

Let c1 and c2 be two smooth, closed, convex, planar curves of the same length, each bounding a flat piece of paper. Choose a point p1on c1 and a point p2 on c2, and glue the two curves to each other (according to arc length) starting with p1 glued to p2. The resulting single piece of paper forms a shape in space called a D-Form by Helmut Pottmann, Johannes Wallner, and Tony Wills. The curves c1 and c2join to form a closed space curve c bounding two developable surfaces S1and S2 These authors ask two questions:

1. It is not clear under what conditions a D-Form is the convex hull of a space curve.
2. After some experiments we found that, surprisingly, both S1and S2 were free of creases, but we do not know whether this will be so in all cases.

Advanced D-Forming

To construct more complex D-Forms it is important to have access to a computer program that allows the user to draw the developed surfaces and measure the perimeters, resizing as required. For D-Forms and my product design work I use Concepts Unlimited, a high end 3D modelling programme from Cadsoft Solutions, but this is somewhat over specified for the two dimensional aspect of drawing and resizing the developed surfaces. In common with even the most sophisticated design programmes, CU can only draw the most basic three dimensional D-Forms consisting of two intersecting cylinders. There seems to be no software program that could process two equal perimeter surfaces and emulate the building of a D-Form from them.

Our experiments show that, provided the perimeters of the two surfaces are continually convex, using any pair of *different* start points will make a unique D-Form. An obvious exception is that that joining two circles only produces a flat disk. An interesting experiment is to divide one of the perimeters up into say ten equal lengths and use these divisions as starting points so that nine D-Forms in different stages of rotation are created. It gets more interesting and difficult if concave curves are included in one or both of the perimeters. Many D-Forms are still possible with concave edges but there seems to be some kind of poorly defined boundary beyond which distortion starts to happen as if the surfaces are tying to enter 'negative space'.

Personal and collaborative work

As a product designer interested in geometry, I have occasionally used D-Forming in my work. It is very satisfying to create a D-Form as a mould and cast a perfectly visually balanced object from it. It is the 'rightness' that D-Forms have that particularly interests me. Since I originally conceived the idea some years ago I have developed a reasonably good eye for how to predict a D-Form from its developed surface but they still have the power to surprise me. Figure 7 shows me holding two steel D-Forms.

Figure 7, D-Forms made from sheet steel

Figure 8 shows how D-Forms have been used as part of a bench. The ends of the bench are cast in concrete using a D-Form.
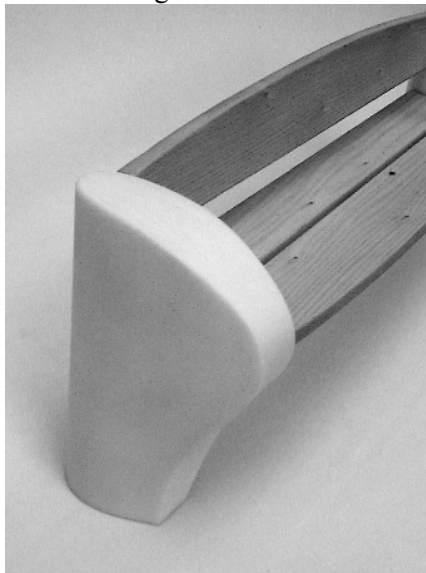

Figure 8, left: street furniture made using D-Forms moulds and right: concrete castings

Intellectual Property

Rather than protect the Intellectual Property inherent in D-Forms, I want the concept to be 'open source'. Please feel free develop the concept independently but all I ask is that you acknowledge me as your source and inform me of your progress so that ideas can be shared.

Summary

Although I discovered D-Forms, the mathematics of them remain much of a mystery to me - if you have some insights do please contact me. In particular, I would very much like to be able to draw the developed surfaces, select the first pair of contact points and then have a computerised model predict the three dimensional form. This is something that CAD programmes simply cannot do (although some boat building programmes can unfold hulls). John Sharp has included some of the mathematics in his book on D-Form but this is just a start. The limits to D-Forming remain unclear partly because the concept is still developing - if you will pardon the pun. Some D-Forms appear to go beyond a border where the two surfaces resist being joined and I wonder if this has is a three dimensional analogy to negative numbers.

What, I do know for sure is that D-Forms are almost always beautiful, curious shapes that deserve much deeper research and understanding. To repeat Prof. Helmut Pottmann's comment, D-Forms are 'an open problem'.

References

[1] Helmut Pottmann and Johannes Wallner. Computational Line Geometry.Springer-Verlag, 2001.
[2] Paul Bourke has some http://astronomy.swin.edu.au/~pbourke/surfaces/dform/
[3] See the D-Form street furniture on the Wills Watson + Associates web site, streetscape products page.
[8] John Sharp, D-Forms, Tarquin 2006

Links can be found here: www.wills-watson.co.uk