Illuminating Chaos – Art on Average

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Abstract

At first sight, chaos and structure seem antithetical. Yet there is an intimate connection between randomness and structure. In this talk we explain some of the ideas we have used for creative artistic design that depend on results from the study of chaotic dynamics. Our intention is to avoid the Platonistic perspective that the role of the mathematician is to dig out and discover the beauty hidden within the mathematics. Our view will be more that of an engineer. How can we use mathematics in a creative way to produce aesthetically pleasing art? (as opposed to 'pretty patterns'.) How can we achieve the effects we want to emphasize in a particular design? We illustrate the talk with examples of (symmetric) designs, many of which have appeared in art exhibitions in the Americas and Europe. As well we give some visual demonstrations and explanations of chaos and, if there is time, indicate some practical applications of these ideas to teaching art students (some mathematics) and mathematics teachers (some art).

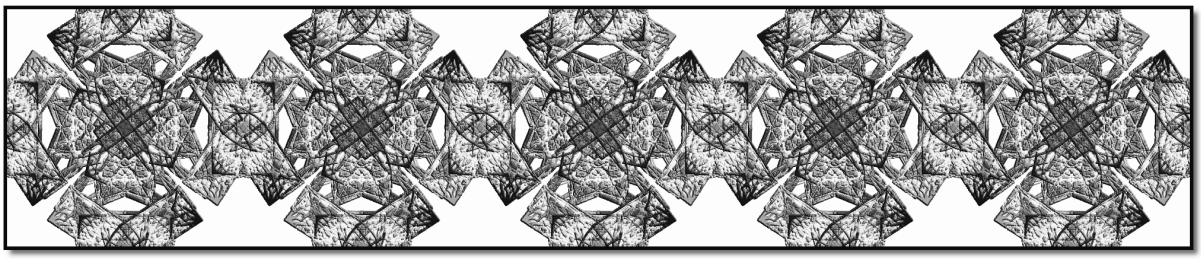

Figure 1: A chaotic frieze pattern of type **pm′m′2**

1 Background notes on the presentation

Associations between art and mathematics are most often tied to geometry and symmetry. This is seen in the way geometry, especially symmetry, is used in design, and in the intrinsic beauty of many geometric objects in mathematics ranging from Platonic solids, through minimal surfaces (soap bubbles), representations of singular algebraic surfaces and more recently a whole new range of visually attractive mathematical objects, such as Julia sets and fractals. However, there is a feeling among many mathematicians that their subject is one of discovery rather than creation. This viewpoint seems antithetical to that of an artist or painter and perhaps this difference in view explains some of the misunderstandings. G H Hardy[1] is quite direct in his views

> "A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with *ideas*. A painter makes patterns with shapes and colours, a poet with words. A painting may embody an idea but the idea is usually commonplace and unimportant. . . "

[1] *A Mathematician's Apology*, Cambridge University Press, Cambridge Paperbacks, 1993.

Much has changed since Hardy wrote these words in 1940 – indeed, one interest of the essay is the many statements it contains that are just flat out wrong. However, if there is to be – at some level – a synthesis (or even a dialog) between mathematics and art then I believe it important to develop new perspectives on the way mathematics can contribute to art (and conversely).

Mathematics is not just an extension of the work of the Greeks on geometry and number theory. The last two hundred years have seen the rise of statistics and probability theory in mathematics and science. It is quite remarkable how many of the basic laws of physics are framed in statistical terms. The same is true of biology, through the laws of genetics, and evolution, through the effects of random mutations. Although it sometimes seems intensely paradoxical (not to mention disturbing), the enormous variety of shape and form we see around us appears to be a consequence of statistical irregularities rather than in spite of them. Without the statistics we would probably be living in Laplace's deterministic and inevitably dull clockwork universe[2]. Statistics and probability are often thought of as the ugly ducklings of mathematics. Look closer and you will see the swans.

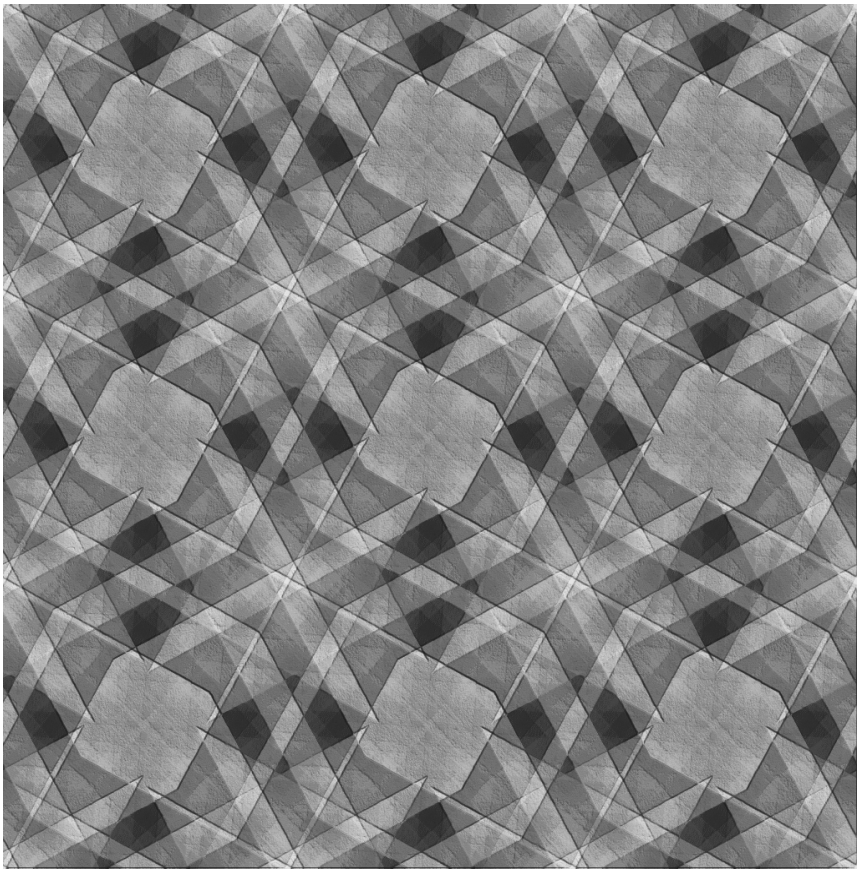

Figure 2: A chaotic pattern of type **p4**

[2]Of course, this is grossly unfair to Laplace, the founder of probability theory. Although responsible for the famous quote on determinism, he also observed that "It is remarkable that a science which began with the consideration of games of chance should have become the most important object of human knowledge" (Théorie Analytique des Probabilitiés (1812))