Cosmati Pavements: The Art of Geometry

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Abstract

This paper considers some pavement designs of the group of artists known as "The Cosmati". We argue that their designs are primarily concerned with conveying the aesthetic of beauty, and yet still have deep implications in the study of geometry. We suggest that this subtle synthesis of form and functionality is essential to the subject of geometry, not merely as an analytical discipline, but as an evolving understanding of space and line.

1. Introduction

This short paper discusses the Cosmati, a group of artists active in Rome, roughly between 1200 and 1300. Their work is of interest to the theme of this conference because it may be said to combine art and geometry at a profound level. In the modern mathematical mindset, geometry is increasingly perceived as a purely analytical discipline, in which aesthetic ideas, i.e., ideas concerning our intuition as to what is, beautiful and harmonious, play a minor or irrelevant role. This view is also reflected in disciplines such as architecture, where the question of the interaction between geometry and aesthetics becomes the question of the interaction between functionality and form. Again, we see a modern tendency to prefer functionality over form in the construction of buildings. To some extent, we are encouraged to judge a building as "beautiful" if it is well constructed, i.e., a high degree of technical skill has been involved in its design and realization.

It is not my intention to condemn this type of thinking. Functional or analytic considerations can and have led to many refinements in aesthetics. Our understanding of art has, in some ways, been increased by the development of science. Good examples can be found in the work of Escher, whose studies in mathematical symmetry could be said to have led to an entirely new art form, or fractal art, invented by Mandelbrot from investigations into complex analysis. However, I wish to argue that in the medieval mindset, the interaction between aesthetics and geometric ideas is more subtle and ambiguous. That is, one can find a two-way interaction between these disciplines, which leads to advances both in art and in the study of geometry. The Cosmati are particularly relevant in demonstrating this theory. They were primarily artists, interested in conveying the aesthetic of beauty; indeed the name Cosmati, translated from Greek, means "the beautiful". However, in their pavement designs they translated their aesthetic ideas into forms that are highly relevant to an understanding of geometry in the true sense of the word, as an evolving and synthetic understanding of the representation of space and line. This is why I have described their work in the title as the art of geometry rather than the geometry of art.

Before giving specific examples of their work, let me give a brief description of the history of the Cosmati artists. The Cosmati were a family, headed by Cosmatus, who is known to have had four sons, Jacobus, Petrus, Iohannes, and Deodatus, all of whom continued in their father's profession as marble workers or "marmorani romani", as they were also known. However, there were other contemporary artists such as Laurentius, Vassallettus, and Drudus de Trivii, working in the 13th century, and artists from the 12th century, such as the Paulus and Rainerius families, who were also known as the Cosmati, due to the similarity in the style and execution of their work. The Cosmati were more than just designers of pavements, they were also architects, responsible for many of the campaniles attached to Roman Churches, such as Santa Maria in Cosmedin and San Giorgio in Vellabro, according to [15], as well as some of the most beautiful cloisters in Italy, such as Subiaco, San Paolo Fuori di Le Mura, and San

Giovanni in Laterano (Figs.1,2,3). They also designed triumphal arches, choirs, screens, porticoes, ambones, ciboria, candelabra tabernacoletti, episcopal thrones, altars, and tombs. A good overview of this type of work can be found in [5]. According to a recent survey in [1], 91 percent of what the author refers to as genuine Cosmatesque production is found in central Italy, and 46 percent can be found in Rome. Some notable exceptions are in Sicily and Westminster Abbey in London. The underlying historical factors that governed the huge output in this area of Italy are complex and interesting in their own right.

Figure 1: *Subiaco.*

Figure 2: *San Paolo Fuori di Le Mura.*

I wish to consider, primarily, the geometric designs used by the Cosmati in their pavements. This study is complicated by the fact that some of the original pavements are missing, either as a result of structural damage over the centuries or extensive restorations during the Renaissance and the 19th century. In some exceptional cases, the designs of the restored pavements are known to be nothing like the original, and in other rare cases, there are even conflicting opinions as to whether the pavements have been restored at all.[1] A good survey on the authenticity of Cosmati pavements can be found in [3]. However, the pavements that I will consider are mostly original, and, at worst, preserve their original geometric designs.[2]

Figure 3: *San Giovanni in Laterano*

Although many variations exist, there are a number of basic motifs that recur frequently in Cosmati designs. The most important of these are the guilloche and the quincunx. The guilloche design can be seen in Figure 4 taken from Santa Maria in Trastevere, Rome. It consists of two interweaving threads that run along the central section of the nave. This braiding image is repeated in the architectural device of twisting columns, which we saw in the pictures of Cosmati cloisters, and also in a number of Paschal candelabra. This makes it clear that the image is symbolic of the resurrection of Christ. It can also be seen as representing the image of the River of Life, from Revelations, an observation that is made in [16]. The quincunx design can be seen in Santa Maria in Cosmedin, Rome, and also at Westminster Abbey (Fig.5). It consists of a central roundel and four smaller surrounding eyelets, around which a single thread is intertwined. In [1], we are given the following description of the symbology of the quincunx:

> "A 2-dimensional abstract representation, that is, the monogram or coded representation, that signifies the 3-dimensional reality associated with the medieval Christian cult."

[1] For example, at Santa Maria in Cosmedin, according to [1], the pavement is original but according to [8], the upper part was transferred to the presbytery sometime during the 18th or 19th centuries.

[2] In cases of doubt as to the authenticity of the geometric design of a pavement, I take the view that, unless the whole pavement was replaced, which is unlikely, the original design was maintained.

Figure 4: *Santa Maria in Trastevere, Rome.*

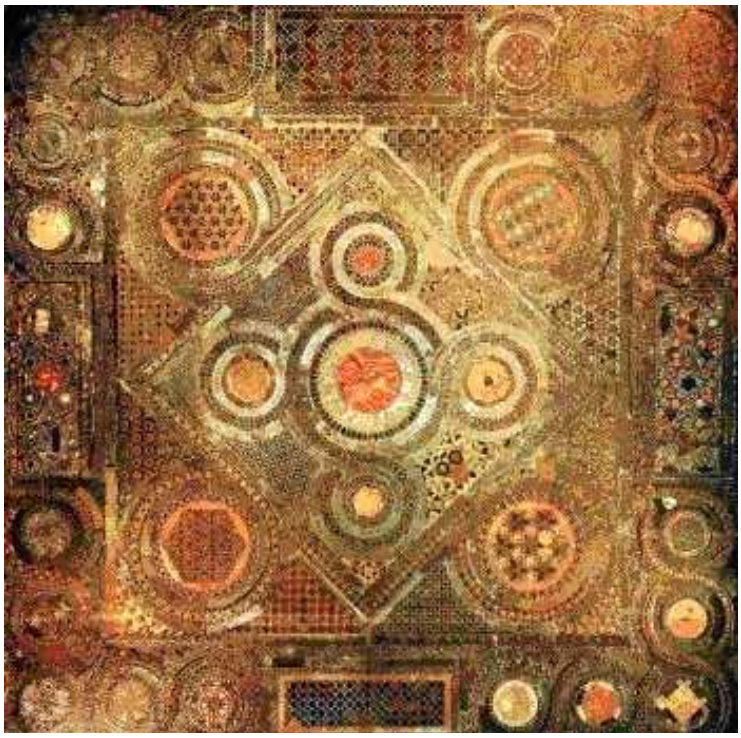

Figure 5: *Westminster Abbey.*

This 3-dimensional reality is the tetramorph, consisting of the 4 hybrid beasts, which represent the evangelists, surrounding either Heaven or the symbolic representation of Christ as a circle. This symbolism was also used by Romanesque architects in the form of wheel-rose windows, for example at Tuscania (Fig.6), in which the central circle is replaced by a wheel, symbolic of Ezekiel's vision from the Old Testament.

Figure 6: *Tuscania.*

Aside from the symbolism, in both the guilloche and quincunx designs, we can see a diversity of aesthetic expression. Whereas the guilloche conveys a sense of femininity, fluency and twisting, the quincunx is more concerned with the classical idea of harmony. In both cases, there is also a suppressed aesthetic of light; the designs are projections of 3-dimensional forms, manifested by the fact that the Cosmati artists left spaces at the points where the represented curves intersect. This aesthetic is further suppressed by the use of dark colors for the marble roundels.

Figure 7: *San Lorenzo Fuori Le Mura.*

In the following design from San Lorenzo Fuori Le Mura (Fig.7) we see a combination of the quincunx and guilloches. The aesthetic effect here is one of dynamism, not accomplished by the quincunx or guilloche form on its own. This aesthetic appears as a more intensified form of the feminine aesthetic of the guilloche, and clearly breaks with the rules of classical harmony. Its historical origins can probably be found in the numerous spiraling designs of Byzantine mosaics found in Rome.

Finally, I wish to consider this design from San Cataldo in Palermo (Fig.8). This is extremely atypical of the Cosmati, in that it is primarily composed of straight lines rather than curves, which conveys a more masculine aesthetic of linearity and fragmentation. The design is clearly influenced by previous Arabic work in Sicily, but due to its suppressed 3-dimensionality, which we observed earlier and which is not found in Arabic patterns, I prefer to consider it as belonging to the genre of Norman art.[3]

The sophisticated aesthetics used by the Cosmati fit into a system that the interested reader can find on my website, at http://www.curveline.net . It is based on four distinct categories of aesthetic judgment, which I in no way claim to be exhaustive. The question of whether we are able to make these aesthetic distinctions seems to be an important psychological question, and is instrumental in deciding whether aesthetics can play a role in the development of geometry as more than a purely analytic discipline.

I wish to finish this paper by considering the geometric implications of this categorization. I can do this more effectively in the framework of algebraic curves, which has been my main area of mathematical research. The effect of aesthetics on geometry is, primarily, to find new methods of representing curves, either in a plane, as was the main concern of the Cosmati, or in 3-dimensional space, which is also evident in Cosmati work. This does not seem to be an analytic question, since, mathematically, the same curve could have a number of different representations or visualizations. Let me consider how each aesthetic category can be and has been used in the study of algebraic curves.

[3] The Normans were at that time the ruling force in Sicily and were responsible for the development of the aesthetics of linearity and fragmentation in the construction of Cathedrals across England, see my forthcoming book [11].

Figure 8: *San Cataldo in Palermo.*

(i). The aesthetic of fragmentation and linearity is clearly evident in Newton's work on the classification of plane cubics [10]. For Newton, a cubic curve should be represented as the fragmented union of three lines, which he defined using his invention of the power series. The algebraic formulation of this idea is usually known as Newton's Theorem. In more modern mathematics, it can be found in the formulation of the Severi Problem, the question of whether a plane algebraic curve can be degenerated to a series of lines in general position.[4] This problem was solved recently by Harris in [4], and depends critically on modern algebraic techniques, such as deformation theory (see for example [14]). This intuition of the fragmentation of a curve into lines was inherent in the Norman development of rib vaulting and underlies the Cosmati work in Sicily.

(ii). The aesthetic of light was used extensively in the development of perspective by artists such as Piero della Francesca [2]. Francesco Severi developed these ideas in his work on the conic projections of algebraic curves and birationality. It leads to the theorem that any algebraic curve is birational to a plane curve, with at most nodes as singularities, a feature clearly evident in the Cosmati representations of plane curves. You can find a fuller mathematical discussion of Severi's work on conic projections in my paper [12]. The aesthetic of light is also used in the problem of focal degenerations of curves, i.e., degenerations to lines in specific position, for example, to lines that all meet in a given point (work in progress).

(iii). The aesthetic of harmony lies behind Plucker's work on duality and his formulas relating the number of nodes, the genus, and the degree of a plane curve. The quincunx representation of a curve used by the Cosmati, is similar to the one used to define the genus of an algebraic curve, as a sphere with a number of

[4] If C is a plane algebraic curve of degree *n*, and there exists a 1-dimensional family of plane curves of degree *n*, containing C and a union of *n* lines, no two of which intersect in a point, then we say that C degenerates to a series of lines in general position. Although it is known that any nodal curve C (having at most nodes as singularities) can be degenerated to a series of lines in general position, the more delicate question of focal degenerations (own terminology) to specific configurations of lines, generalizing Severi's construction of degenerations to lines all meeting in a given point, is still unresolved, see [7] for some recent work on this problem.

attached handles. For a more mathematical discussion of ideas related to duality and Plucker's formulae, see my paper [13]. For a discussion of ideas related to genus, see [6].

(iv). The aesthetics of fluency, dynamism and spiraling have a geometric interpretation in the case of real plane analytic curves. Fluency is a feature of such curves having no nodes, but only a series of inflexions. A spiraling aesthetic is conveyed by such curves with no inflexions, and dynamism is an aesthetic conveyed by curves with an alternating sequence of nodes and inflexions. These ideas lie behind the representation of an algebraic curves in which its inflexions are in positions distinct from the nodes [12]. They can be further developed using the work of Moishezon and Chisini on braid monodromy, the theory of how the geometric structure of an algebraic curve can be understood using braids, that may be considered as instances of real plane analytic curves, or the threads of Cosmati designs. The effect of re-braiding these threads can be interpreted in terms of Dehn twists, fundamental to the idea behind braid monodromy [9]. The geometric puzzles posed by designs such as the one at San Lorenzo, and others, for example at the Sacro Speco in Subiaco, show how the Cosmati were genuinely exploring these ideas at an intuitive level.[5]

The achievement of the Cosmati is extraordinary in that these geometric ideas are explored in their pavement designs. They demonstrate a balance between aesthetics and geometry, which, I believe, is not paralleled in our modern understanding of the subject. The development of our comprehension of this synthesis, therefore, seems to be an important subject of future investigation.

References

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[5] The effect of switching the sequence of nodes and inflexions of Cosmati braids can be interpreted in terms of a sequence of (ushiro) mawashi-geri moves (own terminology). Dehn twists may be understood geometrically by projecting an algebraic curve onto the complex projective plane and seeing the effect of the twist on the braid group. In this sense, an aesthetic of light is implicitly used. However, (ushiro) mawashi-geri moves cannot be usefully interpreted in this way.