Using Polyhedral Stellations for Creation of Organic Geometric Sculptures

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Abstract

We describe the process of making metal sculptures starting from geometric constructions based on stellation of polyhedra. Symmetry transformations and various algorithms of mesh subdivision are used to convert straight edges and planes of stellated polyhedra into smooth organic shapes.

Stellation of Polyhedra

Stellation is a process of extending the faces of a polyhedron in all directions to infinity. These extended faces divide the space into convex polygonal cells. In general, any subset of these cells may be called a stellation of the original polyhedron. Typically, only the most symmetric subsets are considered [1], but stellations with lower symmetry [2] are interesting as well. Figure 1 shows the steps of making the first stellation of icosahedron by attaching triangular pyramids to each face of the original icosahedron. Figure 2 shows the final step of this process - first stellation of icosahedron.
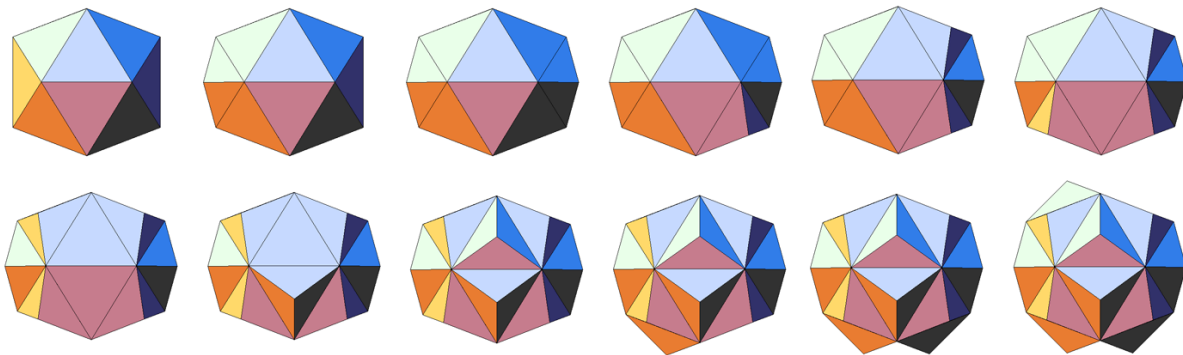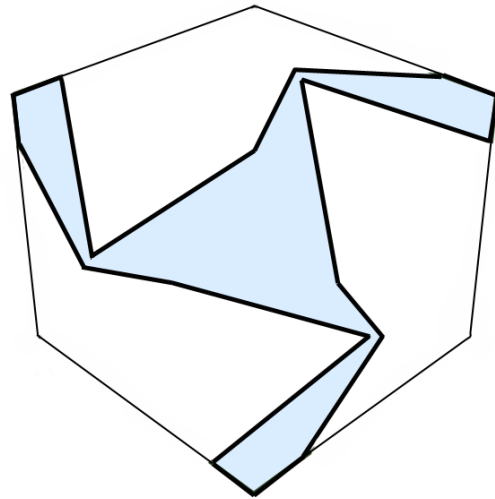

Figure 1: *First steps in stellation of icosahedron. The final shape is shown in Fig. 2*

The calculation of intersections of extended polyhedron faces is a tedious process to do by hand, but it can be easily done using interactive computer programs such as Stellation Applet [3] or Great Stella [4]. One can view the first stellation of icosahedron in Fig. 2 as a combination of the original icosahedron and 20 triangular pyramids attached to each of its 20 faces. However, we adopt here a different view: it is a surface consisting of 20 identical convex flat hexagons. One such face is marked with the bold outline in Fig. 2. These faces intersect along the edges of the original icosahedron and meet each other along the side edges of the triangular pyramids. We would like to build a surface with the same overall topology, but which has no intersections between the faces. One way to construct such a surface is to make deep cuts in each hexagonal face as shown in Fig. 3.
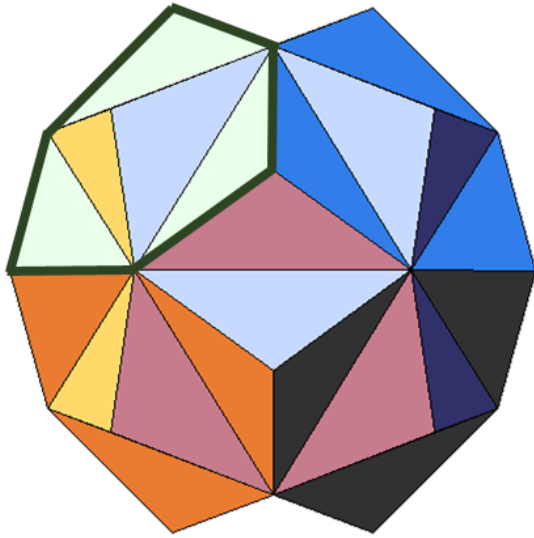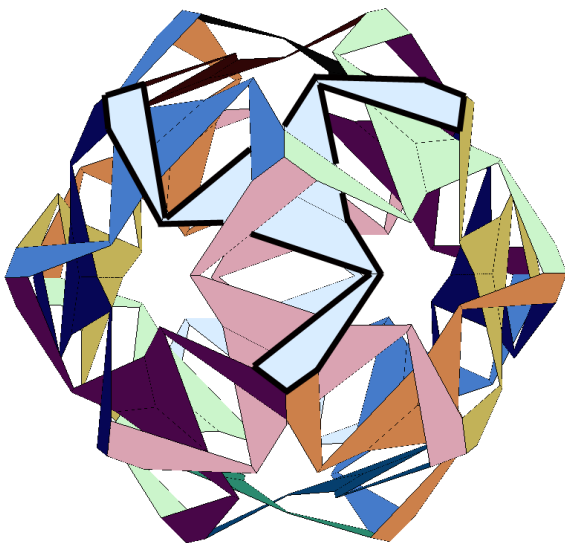
Figure 2: *Complete first stellation of icosahedron with one hexagonal face outlined.*
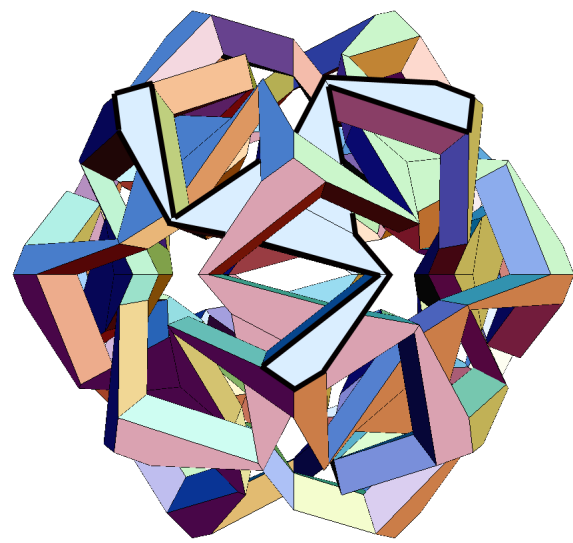

Figure 3: *A face of the first stellation of icosahedron with deep cut-outs.*

The complete surface composed of such prepared faces is shown in Fig.4. In order to make a real solid object one can assign to that surface some finite thickness as shown in Fig.5.

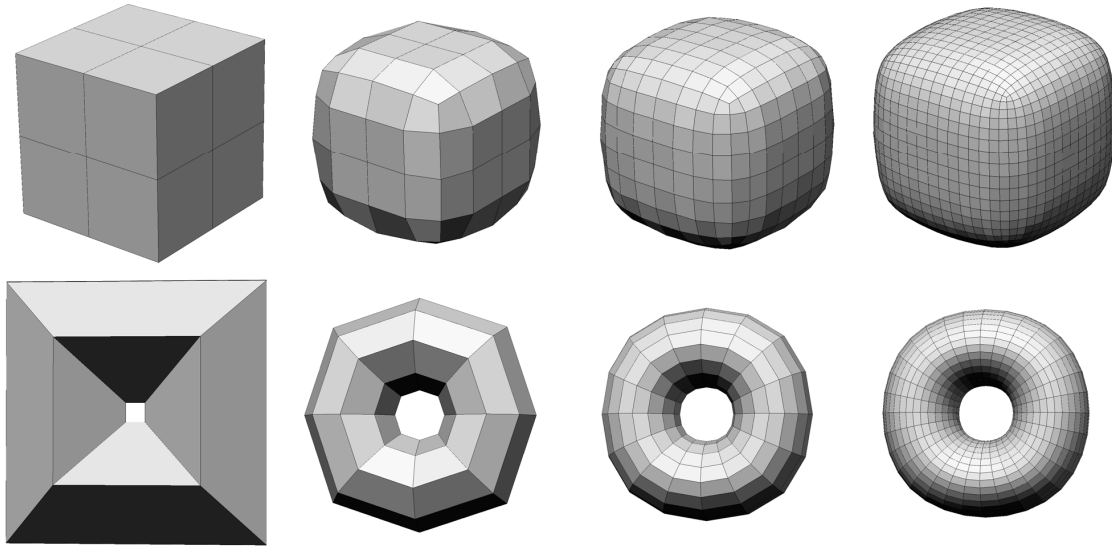**Figure 4.** *The first Stellation of Icosahedron with cut out faces.*

Figure 5: *The faces are given some thickness.*
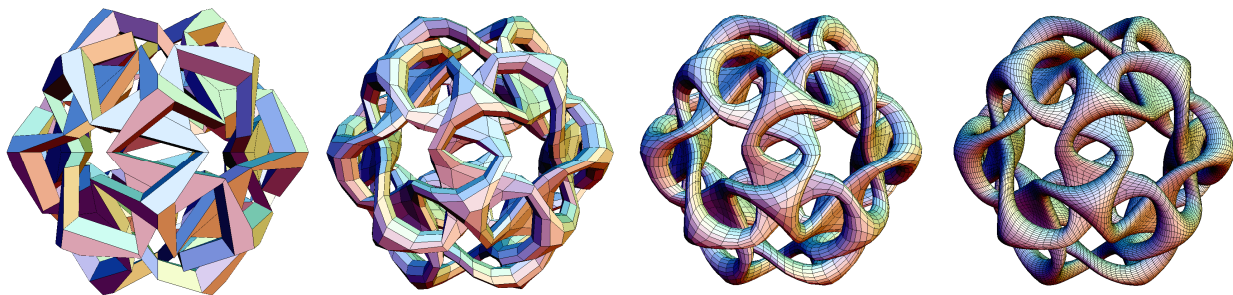
Subdivision

The object in Fig.5 possesses multiple sharp edges, and as such is far from our goal of an organic-looking shape. However, one can apply well-developed computer graphics algorithms known as surface subdivision, to smoothen the surface [5,6]. These algorithms refine a low-resolution mesh into a high-

resolution smooth mesh. The general idea of subdivision is to replace each polygonal face of the existing mesh with several smaller polygons, and then shift the vertices to new positions to make the new mesh smoother. The procedure is repeated several times until the surface is smooth enough. The result of this algorithm is shown in Fig. 6 for two simple initial shapes: the cube and a toroidal shape. One can observe that just a few subdivision steps result in very smooth looking surfaces.

Figure 6: *Examples of mesh subdivision applied to the two simple shapes.*

Application of the subdivision algorithm to the first icosahedron stellation of Fig. 5 produces the sequence shown in Fig.7. The object becomes very smooth already after 3 subdivision steps.


Figure 7: *The subdivision sequence for the icosahedron stellation of Fig. 5.*

One more trick needs to be added to the algorithm. Real organic objects seldom have absolutely smooth surfaces. Typically, they have folds and creases of various types. The subdivision algorithms have been generalized to create such discontinuities in surface derivatives [7]. The idea is to mark some edges of the initial mesh as "crease edges" and then modify the shift rules for the vertices that belong to the crease edges. The result of the application of the modified algorithm to the mesh of Fig. 5 is shown in Fig. 8.

Metal Sculptures

There exist several fabrication technologies that can transform a 3D computer model into a real-world metal object. We have used Selective Laser Sintering (SLS) from 3D Systems [8] and Direct Metal Printing from ProMetal [9]. Both technologies make parts from stainless steel powder infiltrated with molten bronze. The first method produces more precise shapes with finer surface finish, but it has problems with larger pieces. They have the tendency to break during late stages of processing. In addition, the second method can make parts with copper and gold. Figure 9 shows a photo of sculpture "*Icosahedron I"* made using SLS technology. Outer corners of the sculpture have been polished. Polishing creates a pleasing combination of rough and smooth surfaces.
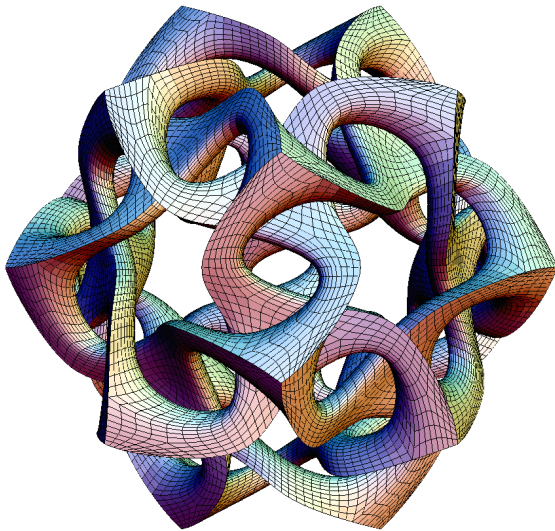
Figure 8. *A subdivision mesh with creases.*

Figure 9. *Icosahedron I.*
Steel and Bronze. Diameter is 4 inches.
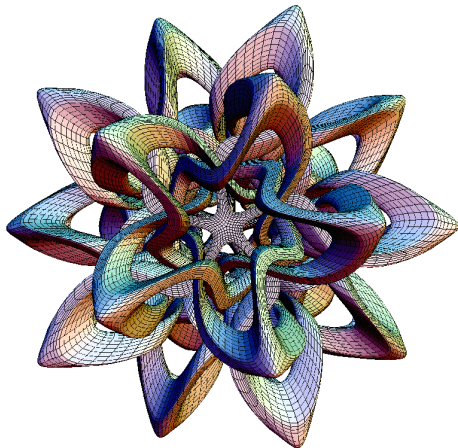
Figure 10. *A subdivision mesh based on great stellated dodecahedron.*

Figure 11. *Dodecahedron X.*
Steel and Bronze. Diameter is 4 inches.

Figures 10-15 show additional examples of author's sculptures based on polyhedra stellations.

Figure 12. *Dodecahedron IX. Steel and bronze. Diameter is 4 inches. Based on Small stellated dodecahedron (the first stellation of Dodecahedron).*

Figure 13. *Dodecahedron IV. Diameter is 4 inches. Based on Great Dodecahedron (the second stellation of Dodecahedron)*

Figure 14. *Rhombic Triacontahedron III. Diameter is 4 inches. Based on stellation of Rhombic Triacontahedron.*

Figure 15. *Dodecahedron IV.* Diameter is 4 inches. Based on Great Dodecahedron.

Conclusion

Modern technology provides an artist with new tools and possibilities to make the art forms he or she wants. However, this raises the question: Can we call objects made with help of computer controlled machines art? Shouldn't art be hand made? Well, art is impossible without use of technology. No "*hand made*" art is truly hand made and modern technology is always used to some degree. The artist in his art work is entitled to use any tools he can get his hands on.

Other artists used stellations in the way similar to the author's. Most notable are M.C. Escher's *Double Planetoid (1949)* and *Gravity (1952)*, and many sculptures of George Hart [10]. The original contribution of the author is application of subdivision algorithms to polyhedra stellations. The author writes his own software in Java and C.

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