The K5 Graph Turned into a Golden Pyramid

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Abstract

This short paper establishes a precise chronological record of a concept I have been dis**play**ing since 2003 in labyrinths and exhibits in one form or another, including in Bridges Conferences. The genesis of the idea is described, theoretical work by friends is acknowledged and briefly explained, and different artwork versions to date are listed and presented as a picture gallery to be consulted more comfortably and with extensions on the Conference CD-Rom. A recent 3-D copy is being proposed for the Mathematical Art Exhibit, and room for further developments, namely on lattices with 'Golden Pyramids', is suggested .

Genesis of an Idea
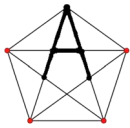
A well-known geometrical figure, I will hereafter name 'The Figure', that I appreciate for years, restricting myself to the beauty of its shape rather than engaging in the many symbolic, esoteric and spiritual aspects it has induced in history, is the pentagram inscribed in a regular pentagon. A highly symmetric object belonging to the 2- and 5-axes symmetries and possessing several instances of the magic (in the sense of geometric perfection) of the Golden Ratio.

A journey in France once led us to spot a sign 'Ronchamp', certainly an emblematic name in History of Art. After a most inspiring time in Le Corbusier's Chapel on top of the hill, an equally unescapable browsing in the art shop (where I also purchased the Master's 'Modulor' texts) suddenly caught my attention to an image in a beautiful book [1] (which I also purchased ...and sadly lost for years, with yet some positive results as personal creative work). The mostly abstract artwork of the main chapel door pinpointed by photographs, was explained by a small line drawing, and The Figure stood there. For a reason I still can't fully explain, I immediately perceived it as **a wireframe pyramid seen from below, its square base featuring its diagonals**.

Before proceeding with the story of this perception, let me stress that as the book recently surfaced, I could revisit carefully pages 58-59, and so I now start wondering if a clue to my vision wouldn't perhaps sit in the photograph of the exterior of the door, something that might have caught me subconsciously, drawing my mind to that 3-D interpretation of The Figure in this sketch by Le Corbusier himself. When looking closer at the left-hand side of the colorful enamel door painting, there indeed is some multicolor opaque object that first looks like a dark tetrahedral pyramid (Fig. 1Z1), but that in fact is another, mirror instance of The Figure, that can suggest a larger pyramid, lying down on a lateral face, and displaying its base with the two diagonals (Fig. 1Z2). I now discovered author Bouvier refers to a text where Le Corbusier says his sketch was inspired by a painting, Le Louvre's anonymous Boulbon altarpiece (with Christ's torso and arms forming an almost regular pentagon). It features The Figure in two mirror images as the *tracé régulateur* of both the source painting and his own on the door (Fig. 1Y). A trapezoidal part of The Figure indeed looks like a square in perspective with the four corners connecting to an upper pyramid top (Fig. 1Z3). How the Boulbon altarpiece induced Le Corbusier's move to construct along medieval lines his painting on the chapel door, how Bouvier's account of it and the placing of the visuals on the page induced my sudden

sight, is an interesting phenomenon indeed, that certainly could justify a doctorate in visual perception psychology... I like to share here two texts the mentioning of Boulbon made me discover: Evans, pp.290-292 [2], and Sarmiento [3], from his ...doctoral thesis, with the *tracé régulateur* curiously here oriented vertically.

Realizing the Idea

This epistemological saga is not yet to stop there and continues to blossom thanks to the existence of a truly seminal construction medium [for those few who don't know it yet, conceived by Baer and Richert, developed by Hildebrandt, Pelletier, Baudoin and Vienne, dignified by Hart, Vorthmann, Richter and Kling [4], to be further happily embraced by many others as a most inspiring thinking tool]. Very excited by above vision, my first move once at home was indeed to construct a Zometool model to substantiate this 'discovery'. Strangely enough I didn't immediately succeed, as no proportions of the square-based pyramid allowed me to get the precise perfectly symmetric shape of The Figure, whatever the inclination or the viewing distance. Then, by chance and some lateral thinking, fiddling with that exquisite tool-toy based on the Golden Section, I ended up building a pyramid with a golden rectangular base, two lateral equilateral triangles and two lateral golden triangles. The base diagonals were modeled with thread, as the 'orange' struts do not yet exist... Surprisingly enough, that model "worked" perfectly (Fig. 1a): indeed, when held in such a way, a short side of the golden rectangle in front, that a lateral golden triangle in the back is viewed orthogonally to the viewing axis through a point above the intersecting base diagonals, and this from the right distance, The Figure appears exactly, as shown using Scott Vorthmann's vZome *Zometool-and-beyond-simulator* in (Fig. 1b1&2) !

Considering the probability of existing information I started searching the web to strangely find only one single source [5], in the esoteric range, the Bibliotheca Arcana, where it is suggested that the Ancients knew this pyramidal view of the pentagram inscribed in a pentagon, as seen from below, and associated the diagonals either with the north/south and east/west directions (indicating they had a square base in mind) or symbolically the five vertices with Earth, Air, Water and Fire, the top being the Spirit...

Beyond the Golden Pyramid

Did anybody else see the Golden Pyramid (and what about Le Corbusier himself ?), I do not know, but Vienne and Bizzarri kindly dug in. The first, who was involved with Le Corbusier in his early architectural life, made nice drawings proving my point (Fig. 1c) as well as an optical montage with his TubeSpace, and the second, first proved it with a geometrical calculation, then upon my request to look for a possible instance where a square based pyramid could still be found to "do" as well, he called me back a couple of days later with a resounding YES-answer: there indeed is another solution with a square base (Fig. 1d), something Vienne again carefully represented (Fig. 1e). Only oddity, though, the top of the pyramid sits among the stars ! Evidently now, looking at it, you can see the wireframe 'Bizzarre Fry', a half infinite square-based prism indeed with its two base diagonals, shooting from close of your eye to infinity.

Mathematical Connections and Room for Further Developments

Only much later did I get aware of some now classical underlying mathematical facts related to The Figure. Graph Theory features the 'complete' K5, a regular non-convex or concave pentagon inscribed in the regular convex pentagon. It is related to the 4-D 4-simplex, or 5-cell, or hypertetrahedron, that orthogonally projects into various 3-D tetrahedra that in turn can orthogonally project into this graph.

Vorthmann and Richter [6] have namely found Zometool- and vZome-constructible 3-D orthogonal projections of the regular 5-cell orthogonally projecting into the K5 graph (Fig. 1f), as Vienne and Bizzarri also readily experimented with, but these don't form square or rectangular-based pyramids (with four coplanar vertices and diagonals). Might the Golden Pyramid be a central projection of some infinity of non-regular degenerate 5-cells ? Open question...

As recalled in next section, I stretched the octa/tetra lattice and materialized half of the octahedra in Golden Pyramids all oriented alike, to carve a 'fractal' Golden Pyramid.

While editing the present paper after referee review, I still found an interesting radial organization of wedges of above lattice developing outward around a 5-axis. It results in central **pentagrams** in half of the alternating tesselation layers orthogonal to the 5-axis, but the K5 is only to be seen (a bit blurred by other struts, though) as central projections from inside points (Fig. 1g1). The 'kernel' or 'seed' of this organization can be considered as a superposition along the 5-axis of adequately stretched pentagonal gyrobicupolas (Johnson Solids J31 with squares and equilateral triangles stretched into golden rectangles and triangles producing an internal arrangement as a stacking of 10 Golden Pyramids, 10 'golden tetrahedral fillers', to indeed grow the above lattice, and 2 pentagonal pyramids with equilateral triangular sides radiating from the center). Skewed transversal triangular and hexagonal planar tesselation developments are also contained. This lattice might merit further thoughts, an extended literature search, and if not yet existing, a theoretical approach studying its special geometrical properties.

Finally, to stay in the Zometool arena, I contributed a vZome 'Golden Kepler's Obsession' (Fig. 1g2).

Conclusion : Realized Artwork

The Golden Pyramid assembled in bamboo sticks with sandows sliced from tire inner tubes was often exhibited, namely, among Zometool sculptures, at Bridges 2006 in London (Fig. 1h).

Also rendered as an image, carved out the above first lattice, using vZome, it was exhibited at the Joint Mathematical Meeting San Francisco in January [7]. A 3-D print of it is proposed for the Mathematical Art Exhibit of this conference (Fig. 1i).

Further see drawings of 'The Snakes Golden Pyramid' (Fig. 1j), 'The Atomium Pyramid' (Fig. 1m) and the one I called 'The Impossible Multibass Golden Pyramid', an *à la Penrose* 'gift' to fellow Belgian artist Jacques Beck (Fig. 1k), where I decided to weave the K5 into a knot (Jacques starts tying knots !) and still highlighting the perspective effect with thickening the "struts" close to the eye. This resulted in parts of multiple double basses (an instrument I once started learning), hence the title, a play of words suggesting Beck's *Multisculptures* resting on different bases, that he is presenting in this conference...

Acknowledgments, Copyrights and References

[1] Yves Bouvier and Christophe Cousin, *RONCHAMP une chapelle de lumière*, Néo éditions, 2005.
[2] Robin Evans, *The Projective Cast: Architecture and its Three Geometries*, MIT Press, 1995, http://books.google.com
[3] Jaime Alberto Sarmiento, *La Puerta de Ronchamp, 1997,* http://upcommons.upc.edu/revistes/bitstream/2099/1917/1/Puerta%20de%20Ronchamp.pdf
[4] Paul Hildebrandt, *Zome-inspired Sculpture*, Proc. Bridges 2006 Conference, London;

http://www.zometool.com

[5]   'Apollonios Sophistes', *The Pythagorean Pentacle*, 1996, 1999,
http://www.cs.utk.edu/~mclennan/BA/PP.html
       and John Opsopaus, *The Pentagram and the Elements*, 1993, http://www.cs.utk.edu/clennan/BA/JO-PE.txt

[6]   Scott Vorthmann, Zome 5-Cell with Ghost Symmetry, http://vzome.com/models/2007/04-Apr/5cell/

[7]   Samuel Verbiese, http://www.bridgesmathart.org/art-exhibits/bridges2007/verbiese.html , Anne Burns, ed.,
       same in 2009 and 2010 with printed versions, Robert Fathauer , ed.

Gallery of Images

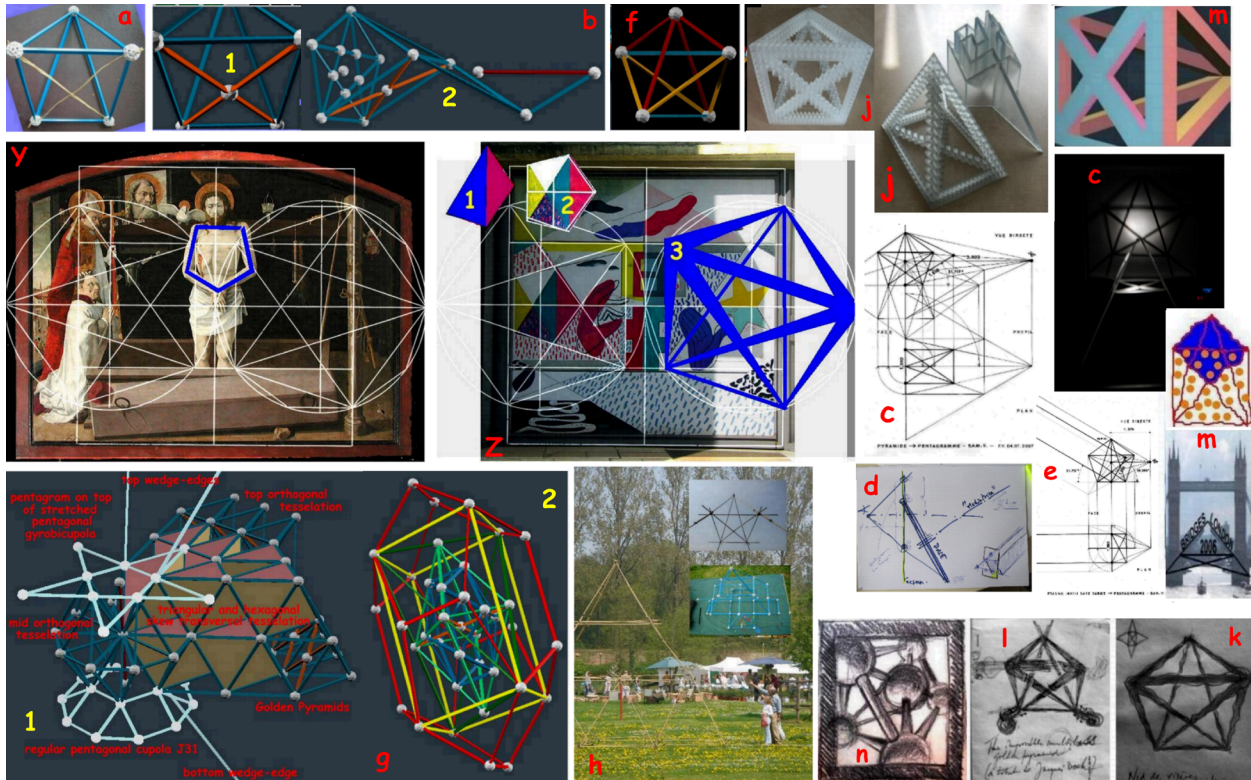Figure 1 : (These and more -larger- images are available on the Conference CD-Rom)

a   The Figure as first Zometool model   **b1**   Same in vZome
b2 Lateral view of the pyramid, the central projection (The Figure), and the viewing axis and distance 'materialized' in vZome
c   Fabien Vienne's confirmation drawing and optical montage, projecting a TubeSpace model on a screen as The Figure
d   Didier Bizzarri's drawing of the 'Bizzarre Fry'   **e**   Fabien Viennes's confirmation drawing of the 'Bizzarre Fry'
f   Zometool vZome 3-D orthogonal projections of the regular 5-cell by Scott Vorthmann and David Richter
g   vZome models of part of the axially symmetric Golden Pyramid Lattice and of the 'Golden Kepler's Obsession'
h   One 'Golden Pyramid' on top of a bamboo octo-tetra lattice structure on a labyrinth with an insert view of The Figure as from
        below (more other 'Bamboo Golden Pyramids' viewed as The Figure, including the one of Bridges London, on CD-Rom)
i   Some Zometool sculpture projects involving the Golden Pyramid, more on th CD-Rom
j   The 3-D print of a somehow 'fractal' Golden Pyramid proposed for the Mathematical Art Exhibit
k   A sketch of "The Snakes Golden Pyramid"   **l**   A sketch of "The Impossible Multibass Golden Pyramid"
m   Other playful works, some seen from other view angles, a.o. cylindrical perspectives, incl. my interpretation of Phillip
        Kent's logo for Bridges 2006, London, itself after Euclid   **n**   A sketch of "The Atomium Golden Pyramid"
Y   A by me annotated superposition courtesy by Y. Bouvier of Le Corbusier's *tracé régulateur* and the Boulbon painting
Z   Another one of the same Le Corbusier's *tracé régulateur* and his chapel south door painting, here highlighting The Figure
        and the possible clue to my vision explained with highlighted inserts