Photographic Fractal Trees

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Abstract

We present several fractal trees created by iterating building blocks constructed from photographs of real trees. The use of photographs of different types of trees, along with variation of the parameters available for construction of the trees, allows a wide variety of forms to be realized. The trees shown have self-similarity fractal dimension varying from 1.45 to 2.17, and infinite series have been used to characterize the number of branches and the area of the trees. Randomization of the construction process is demonstrated to yield less regular and more naturalistic tree forms.

1. Introduction

Fractals are objects that exhibit self similarity on different scales. In other words, repeatedly zooming in on a fractal reveals similar structure over and over again. In a mathematical fractal, the level of detail is infinite. In a fractal object in nature, this zooming in can typically only be carried out a few times, and the similarity is less regular than in mathematical fractals. Examples of fractals in nature include mountains, clouds, coastlines, and arteries in the human body. The last is an example of a branching fractal, in which a feature branches into smaller features repeatedly. Pythagorean trees are simple mathematical branching fractals constructed of alternating triangles and squares [1,2]. Trees in nature are also branching fractals.

Mathematical fractals are created by iteration, in which a step or series of steps are carried out repeatedly. In this paper, we describe a technique for creating fractal trees by iteratively arranging copies of photographic building blocks. The resulting constructions are more fractal than natural trees and can vary in appearance from naturalistic to fantastical. The work described in this paper had its roots in earlier tree-like constructs created by iteratively arranging spirals [3].

All of the photographs were taken using a relatively low-cost digital camera. The drawing program FreeHand was used to create preliminary designs of many of the trees, and Photoshop was used to create the final tree from the photographs.

2. Method for Creating Photographic Fractal Trees

The process of creating the fractal trees can be broken down into four basic steps.

1. Create a preliminary design.
2. Identify the tree or bush that will be used and photograph it.
3. Digitally alter the photograph(s) to fit the template designed in Step 1.
4. Iteratively construct the tree.

For some of the trees, the design was driven by a desired fractal form, in which case the first two steps were in the order shown above. In other cases, it was driven by a photographed tree form, in which case the first two steps were reversed in order.

Designing a fractal tree involves designing a building block and then iteratively constructing a tree from it. Design choices include how many branchings will occur in each segment and exactly where and how the second generation of segments will mate to the first segment. Each segment will be scaled down by some factor, rotated by some amount, and possibly reflected.

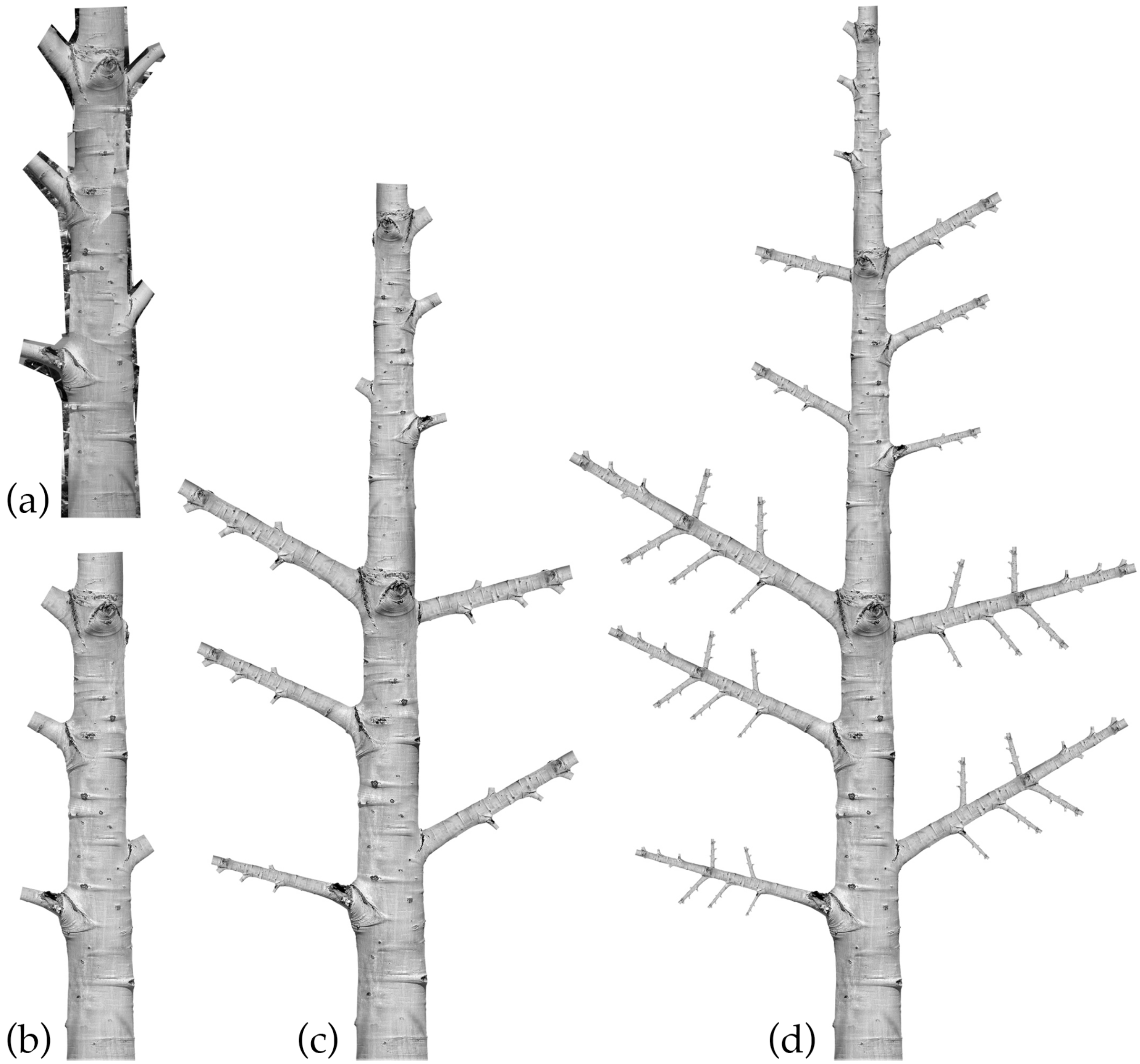

Figure 1: *Example of the construction process for a photographic fractal tree. (a) One or more photographs are combined to create a roughed-out photographic building block. (b) Adjustments are made to allow the different photographs to join seamlessly, and for the scaled-down building blocks to join seamlessly to the larger building block. In addition, the background is carefully trimmed away. (c) Scaled-down copies of the building block are arranged around the original (first generation) building block to form the second-generation tree. (d) Scaled-down copies of the second-generation tree are arranged around the original building block to form the third generation tree.*

An example is shown in Figure 1. In this case, the preliminary design was done before photographs were taken, with the intent of using an aspen tree for the photographic building block. Since it would be nearly impossible to find a real tree that had the desired combination of branch sizes and locations, several photographs were taken of more than one tree. Pieces were then cut out of a few different photographs and pasted together, with some preliminary adjusting of shading and scaling to get the initial version of the building block shown in Figure 1a. Further distortions in shape and adjustments in shading, followed by trimming around the edges resulted in the final building block shown in Figure 1b. This building block is the first generation of the tree, which includes the lowest portion of the trunk.

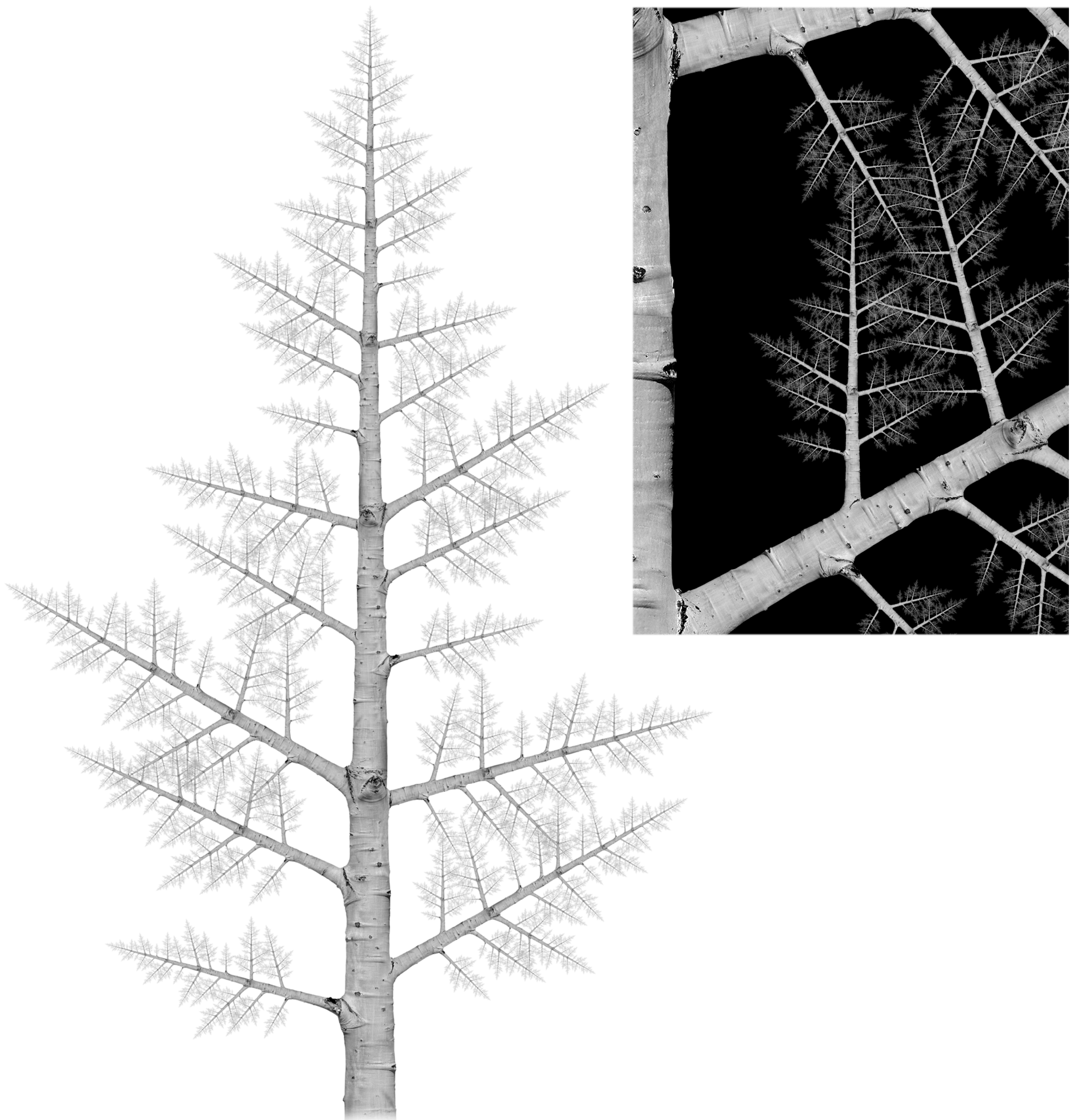

Figure 2: *The final fractal tree that results after 15 iterations of the sort shown in Figure 1. The inset shows a detail of the tree, against a black background for better contrast.*

The next step in constructing the tree was to make six copies of it, to scale, rotate, and reflect (as desired) each one, and then position them at six different locations along the first generation tree. These seven objects were then merged to create the second-generation tree shown in Figure 1c. Six copies of the second-generation tree were then made, and they were transformed and positioned relative to the first generation tree using the same set of transformations to form the third generation tree shown in Figure 1d. This process was continued until the additions to the previous generation were so small as to be insignificant to the eye at the full scale of the finished print, as shown in Figure 2. In this case, 15 iterations were performed in order that the tree looks line an infinitely-detailed fractal even at dimensions of 28″ x 40″. At that size, the height of the building block is reduced to approximately one pixel. In the final print, a black background was used, as in the detail in Figure 2.

3. Further Examples

The different types of trees and bushes that can be used as photographic source material, along with the different choices available in the design of the structures, as described above, allow a wide variety of forms to be obtained using this construction method. In this section, we present a few additional examples. More examples not shown here can be found in other work by the author [4,5].

In Figure 3, a fractal tree full of spiraling segments has been constructed from a photograph of a cholla cactus skeleton. In general, structures that curl in one direction will form when reflections are not employed. If there is sufficient turning in each generation, spirals will result. Note that only two smaller copies of the photographic building block are added with each iteration.

The tree shown in Figure 4 has straight, thin branches that overlap heavily, creating a complex collection of nearly straight-line segments. This tree is not very naturalistic, but emphasizes its fractal character through the boundary of the branching regions. The photographic basis for this tree was a group of twigs from a palo verde tree. The straightness of the twigs and the nearly right angles between the twigs give this tree a distinctive appearance.

The tree shown in Figure 5 is much more naturalistic than that of Figure 4, but its fractal character is still quite evident. The branches roughly form a series of triangles that reduce in size moving from lower left to upper right. Photographs of a royal poinciana tree in Hawaii were used to create the photographic building block for this tree. In contrast to the trees shown in Figures 2–4, for which 15–20 iterations were performed, the construction of this tree was terminated after eight iterations. More iterations were found to muddy the appearance of the tree due to the large amount of overlap of the branches. This has the effect of making the smallest features more naturalistic, as they are similar in size to the smallest twigs on a large natural tree. Note that no reflections are employed in this tree. However, in contrast to the tree of Figure 3, there is not enough turning to one side to allow spirals to form.

4. Mathematical Properties of the Trees

Relatively simple mathematical analysis can be used to characterize the properties of these trees. Issues that can be addressed include the number of branches, the area of the trees, and the complexity of the trees.

If b_i is the number of smaller branches added per larger branch in the ith iteration, then the total number of branches added in the ith iteration, N_i, is given by $N_i = (1)\ b_1\ b_2\ b_3 \ldots b_i$. ($b_0 = 1$, the trunk). For all of the trees shown in this paper, b is the same for each iteration, in which case $N_i = b^i$. For example, if $b = 3$, as in Figure 4, three branches are added in the first iteration, nine in the second, etc. This tree was iterated 15 times, so the number of branches added in the final iteration was 3^{15}, over 14 million.

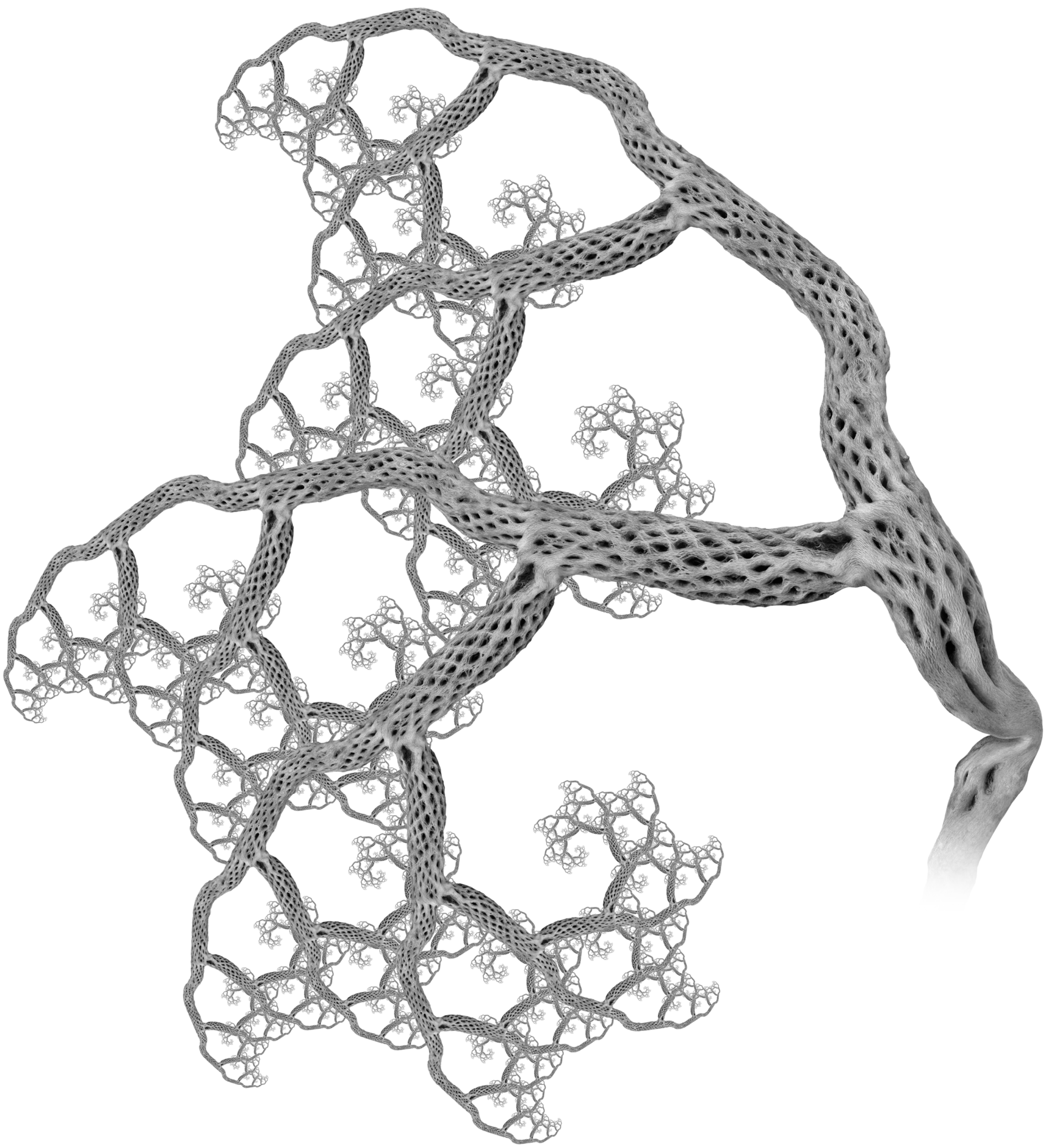

Figure 3: *A fractal tree formed from photographs of a portion of a cholla cactus skeleton.*

The total number of branch segments after i iterations is given by $N_T = N_1 + N_2 + N_3 + \ldots + N_i$. If b is the same for each iteration, $N_T = b + b^2 + b^3 + \ldots + b^i = b(1 - b^i)/(1 - b)$.

The area of one of these two-dimensional trees can also be examined. If the area of the first generation is arbitrarily set to 1, then the area of the branches added in the second generation is the sum of the area of each of the b branches: $A_2 = s_1^2 + s_2^2 + \ldots + s_b^2$. (The area of the first added branch is the area of the first generation, 1, times the square of the scaling factor s_1 for that branch, etc.) For example, the tree in Figure 4 adds three branches with scaling factors of 0.55, 0.60, and 0.66, giving an area of $0.55^2 + 0.6^2 + 0.66^2 \approx 1.10$ for the added branches. Each of these second generation branches will have a similarly larger area added to it in the third generation, so the area of the added branches in each generation increases by the same factor relative to that of the preceding generation. I.e., the total area A_T is given by a geometric series, so $A_T = 1/(1 - A_2)$. For the four trees shown in Figures 2-5, A_2 is approximately 0.83, exactly 0.85, approximately 1.10, and exactly 1.0. As a result, the total area diverges (become infinite) in the limit of an infinite number of iterations for the trees of Figures 4 and 5. By inspection, the infinitely iterated trees clearly fit in a finite area on the page. The infinite area is possible because of the overlap of branches.

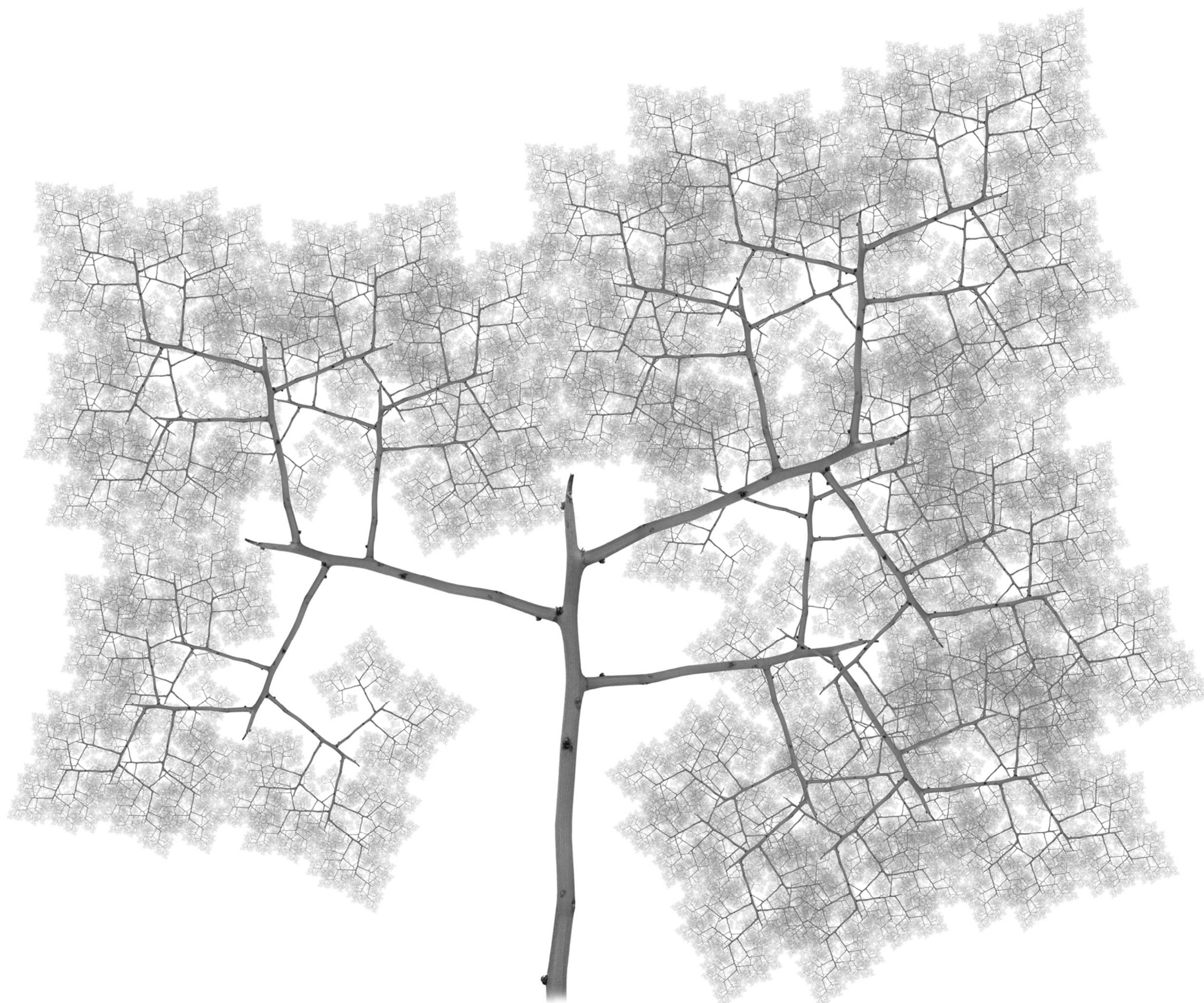

Figure 4: *A fractal tree formed from photographs of palo verde twigs.*

A measure of the complexity of a fractal is provided by the fractal dimension, which evaluates how fast a parameter like length increases as scale decreases. There are several different notions of fractal dimension [1]. For regular structures like those shown here, the self-similarity dimension provides a ready measure. This is given by $D = (\log b) / (\log 1/s)$, where b is the number of pieces into which the structure can be divided, and s is the scaling factor. In this case, b is the number of branches added per larger branch. The scaling factor s is different for each branch, so an average value for the branches added was used for each tree. The approximate values of D calculated for the four trees shown in Figures 2-5 are 1.45, 1.61, 2.17, and 2.00 respectively. Qualitatively, this trend agrees with the amount of overlap observed in the branches of these four trees. Notice that a fractal dimension greater than 2 for a planar structure is only possible with overlapping features.

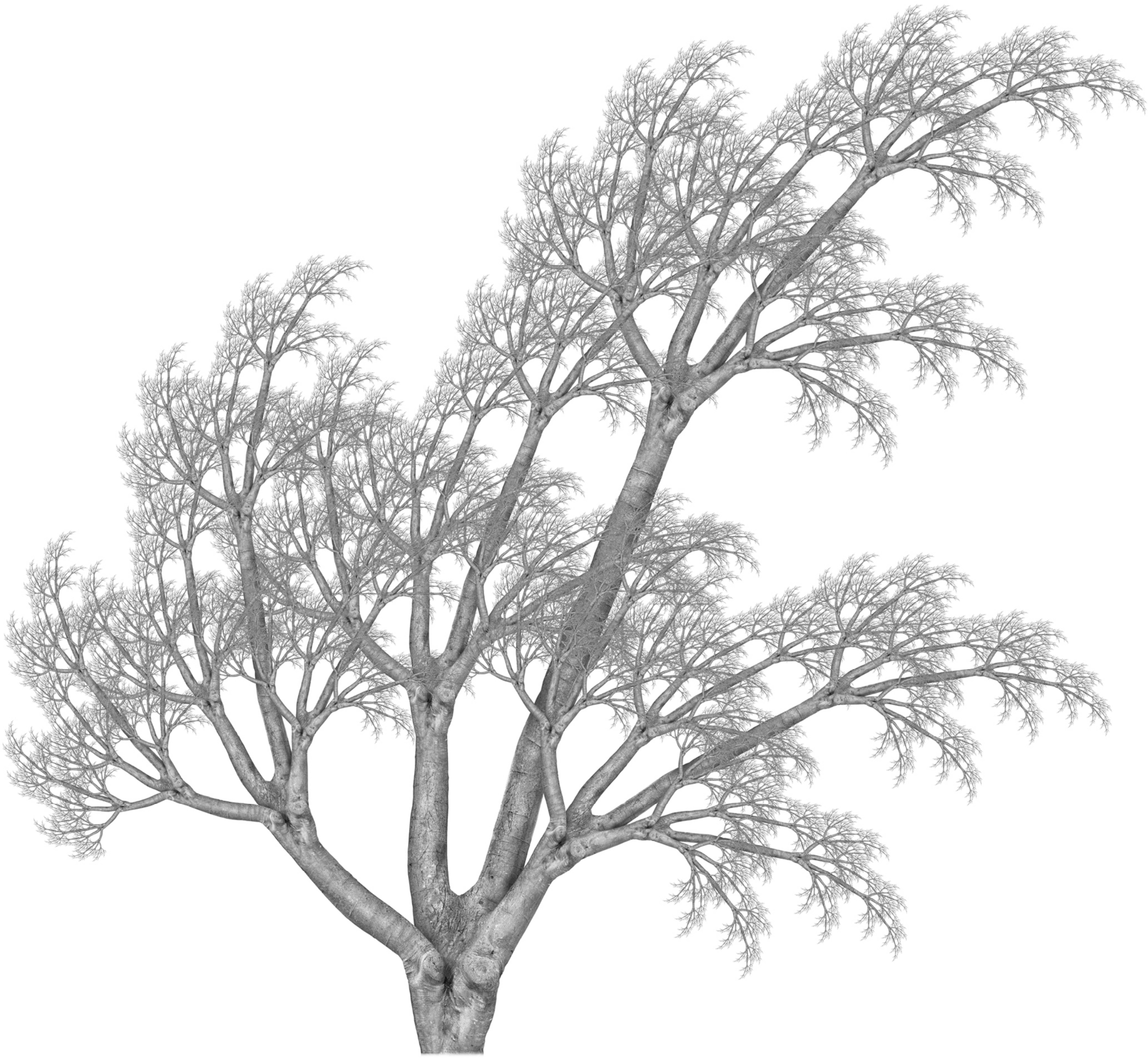

Figure 5: *A fractal tree formed from photographs of a royal poinciana tree. There are no reflections of the branches in the construction of this tree.*

5. Randomized Fractal Trees

For the trees shown in Figures 2-5, the series of transformations carried out was identical for each iteration. However, one or more of the parameters can be varied in order to achieve additional tree forms. Varying parameters in a random manner would be expected to generate less mathematically regular structures, which therefore have the potential to appear more naturalistic.

An example is shown in Figure 6, where the same photographic building block was used as for the tree of Figure 5. For these trees, however, the choice of whether or not to reflect each branch at each step was made randomly. This was accomplished by rolling a 20-sided die at each iteration. The numbers 1-16 were used to determine which branches reflected. (The die was rolled again if 17-20 came up.) The choices can be set by assigning "0" to unreflected and "1" to reflected in the binary representation of the number. For example, the number 5 is 0101 in binary, which can be read from left to right as determining the first and third branches from the left to be unreflected, and the second and fourth to be reflected. With eight iterations, there are 16^8 (over 4 billion) distinct trees that can be formed, three of which are shown in Figures 5 and 6. The two trees shown in Figure 6 are the result of two different sets of eight rolls of the die. Additional examples of randomized fractal trees can be seen in References 4 and 5.

Figure 6: *Two randomized fractal trees created using the same photographic building block used for Figure 5. In this case, the choice of whether or not to reflect each of the four branches at each iteration was made randomly.*

6. Conclusion

We have presented a variety of fractal trees created by iterating photographic building blocks. The use of photographs of different types of trees, along with variation of the parameters available for construction of the trees, allows a wide variety of forms to be realized. The trees shown have self-similarity fractal dimension varying from 1.45 to 2.17, and infinite series have been used to characterize the number of branches and the area of the trees. Randomization of the construction process has been demonstrated to yield less regular and more naturalistic tree forms. Even more naturalistic forms could be realized, for example by varying the number of branches added with each iteration.

References

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