Flying Patterns

Jean-Marc Castera
6, rue Alphand, 75013 Paris, France
E-mail: jm@castera.net

Abstract

Contribution to the investigation on Islamic pentatonic patterns, trying to highlight a multiplicity of points of view.
1. Firstly, I propose a morphogenesis of the Persian style, and give the following applications :
- An analysis of two famous traditional patterns that have yet be studied through different view points. One in Karatay, Turkey (J. Rigby, [10] P. Cromwell [4]), the other in Maragha, Iran (E. Makovicky [7,8], P.J.Lu and P.J.Steinhardt[6]). Where we can see that both are basically the same, coming from the same step in the proposed morphogenesis.
- The "X-puzzle", an application from the first step of our morphogenesis (there will be a workshop at the conference).
2. Then, considerations about the "prototiles" (or "girih tiles") point of view.
3. Last, a relationship is given between "fundamental" and Persian patterns, through a process of decoration. Consequently, definition of a new set of prototiles.
All those concepts can be used by designer to create new patterns.

This contribution belongs to a pattern designer point of view, more than to a mathematical or historical point of view. As Bridges is dedicated to a meeting of Art and Maths, but inherit a scientific format for its publications, please consider this paper as experimental in the sense that the demonstration is given by images more than by words.

1. A Morphogenesis

Five-fold patterns can be found more frequently in the Eastern part of the Arabic world than in the West, but the one pictured in Figure 1 can be seen everywhere. We will call it the «fundamental» pentatonic pattern. A fashionable explanation of Persian-style patterns uses the concept of an underlying isometric network made of «prototiles» (or «Gerih tiles», coming from the famous «Topkapi Scroll», see [9]). But if a pattern comes from such a «protopattern», where does this protopattern comes from? Certainly, if the prototile system was used, it was not the beginning of the story. We must not be amazed by seeing prototiles behind a pattern that uses the specific Persian tile set T1 shown in Figure 8.1, as this is a necessary consequence. That is, we have the age old problem of the chicken and the egg.
The amazing thing is the high level of relationship between all those patterns, and the various approaches possible.

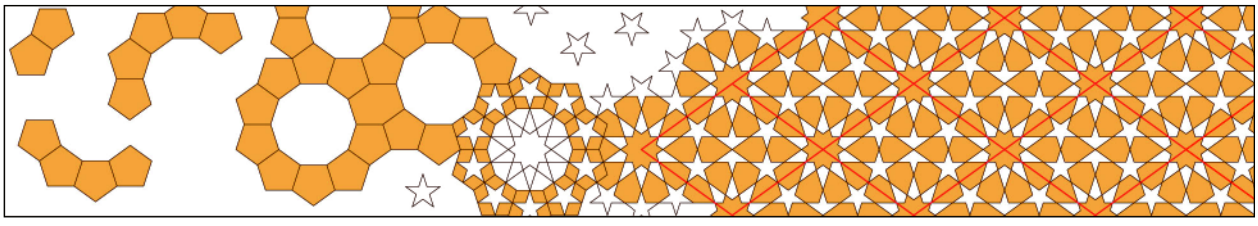

Figure 1: *Prelude, the «fundamental pattern», coming from the dance of 10 pentagons. It can also be seen as made of decorated Penrose rhombi (right part).*

Now take a look, for example, at this famous pattern from Karatay (fig. 2):

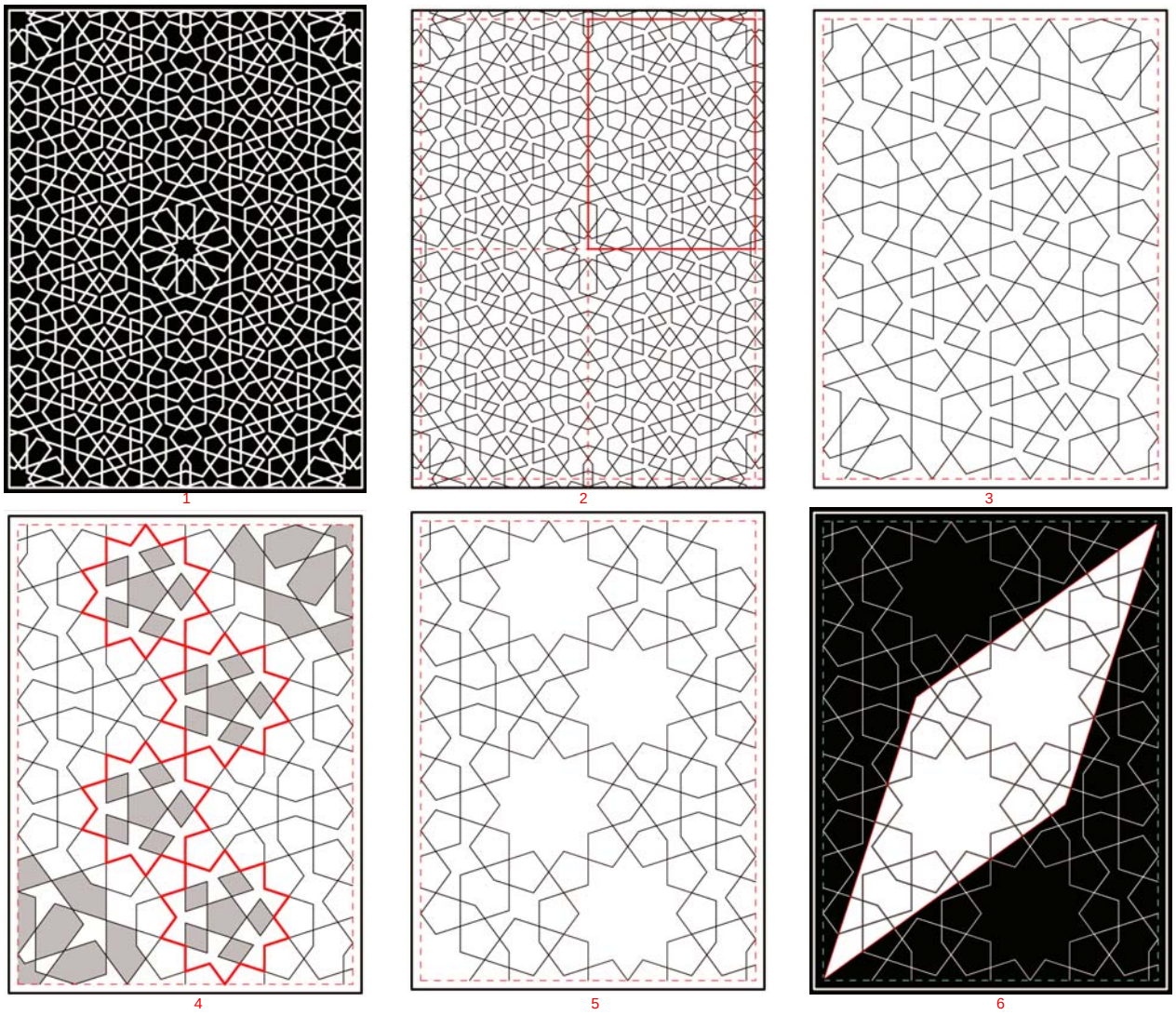

Figure 2: *A point of view on the pattern from Karatay. This pattern has already been studied by J.Rigby [10] from another point of view. I have also another analysis that cannot be given in this short paper.*

Firstly, get rid of the width of the line (2.2), then consider just a quarter part of the motif (2.3). Now, we are going to make two more simplifications (2.4, 2.5): remove the tiles inside the big stars, and replace the tenfold stars at the two opposite corners by more simple tiles. We can now consider the initial Karatay pattern as just a variation from this basic pattern.
Now, at this point it is noticeable that there is a Penrose small rhumb that matches exactly inside the rectangle (2.6). Couldn't we say that the whole pattern is part of a bigger one, made of decorated Penrose rhombi ?

So comes the idea of a morphogenesis : we are going to design a serial of decorated small Penrose rhombi, from the simplest one (the smaller) to more complex ones. For each solution, we have to pattern -if possible- the associated large rhomb. At the complexity step n, a couple [VnS (small rhomb), VnL(large rhomb)] define not only one pattern but a family Vn, because the rhombi can be put together in different arrangements (periodic or not, see fig.3), and because of the possible local variations.
Rules for the tiles : continuity of the line, all angles multiples of 36°.
In this system, the pattern above belongs to the family V4 (fig.3).
A funny thing is to see that the famous pattern from Maghara belongs to the same family (fig. 5) !

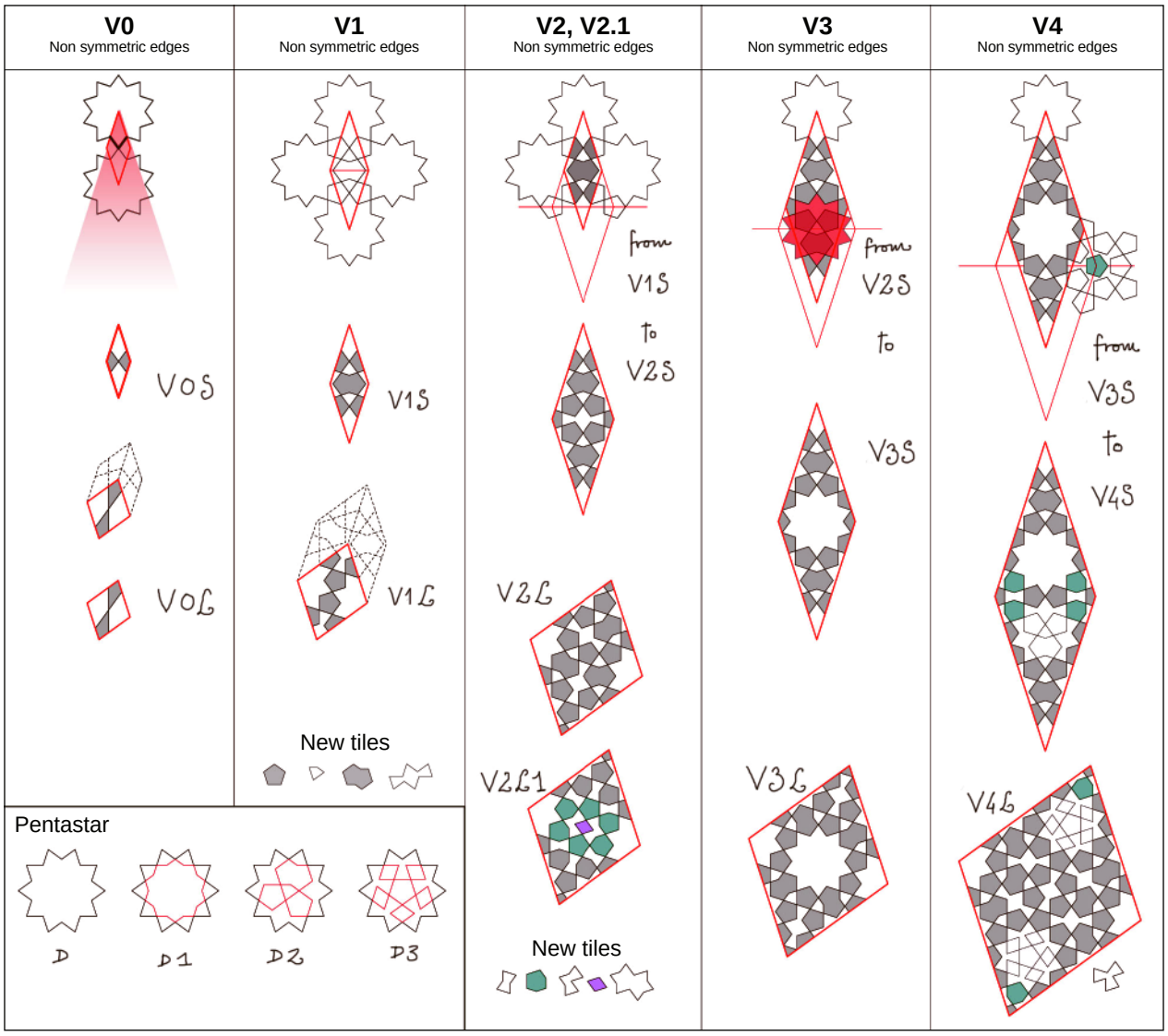

Figure 3: *A morphogenesis. Bottom left, the possible decorations of the "Pentastar", D.*

We call "Pentastar" the star (D) we are starting from. It is centered at the top of a 36° sector. Note that it could be rotated by 18°, giving way to an other family that cannot be include in this short paper...
At the step 2 we have two solutions, V2L and V2L1, for the large rhomb. So, two families of patterns, V2 made of V2S and V2L, and V2.1 made of V2S and V2L1.
Of course, it is easier to set up the rhombi together when the edges are patterned in a symmetric way.
But, what about **V0** ? Back to it later (fig. 7)...

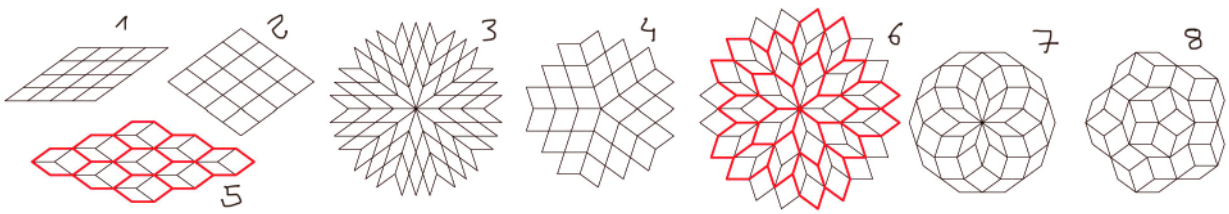

Figure 4: Structures. *1-4 needs symmetric edges. 1-6 are periodic or radial. 7-8: towards non periodicity.*

Application 1, from the family V4: Analysis of two famous traditional patterns

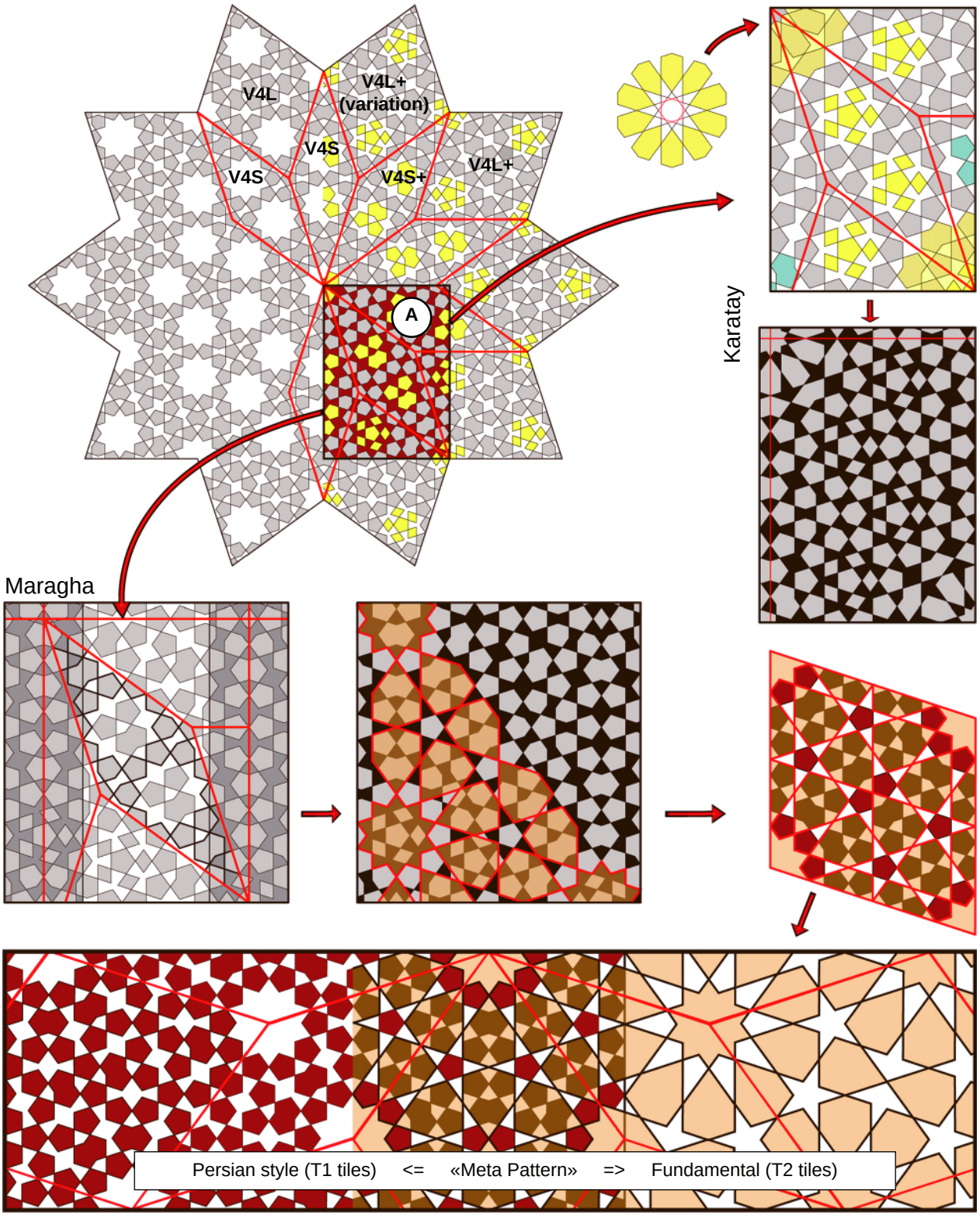

Figure 5: *Pattern from the V4 generation. Into the rectangular window (A) is the structure of the Karatay pattern (1/4 panel, see more in fig.2) and of the Maragha one as well. Bottom: In another view point, Maragha pattern can be considered as coming from a decorated fundamental pattern (see fig. 8.4).*

Application 2, from the family V0: The “X-Puzzle”

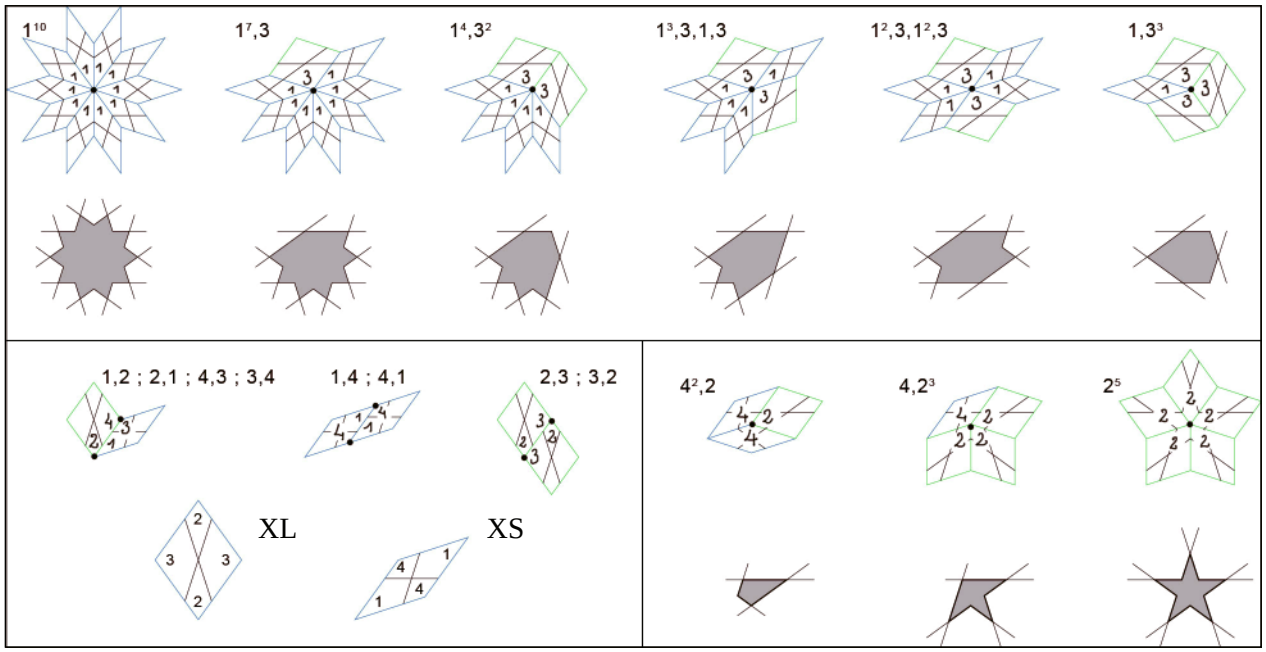

Figure 6: *This puzzle is made from the two Penrose rhombi decorated with two X-shaped lines, XL and XS, coming from the V0 generation (fig. 3). Bottom-left, the impossible arrangements (the lines are broken). Because each angle is a multiple of 36°, we note 1 for 36°, 3 for 72°, etc... In gray, the complete set of tiles drawn by the X-lines. Let “X-Pattern” be the pattern drawn by the X-lines when the X-tiles are put together.*

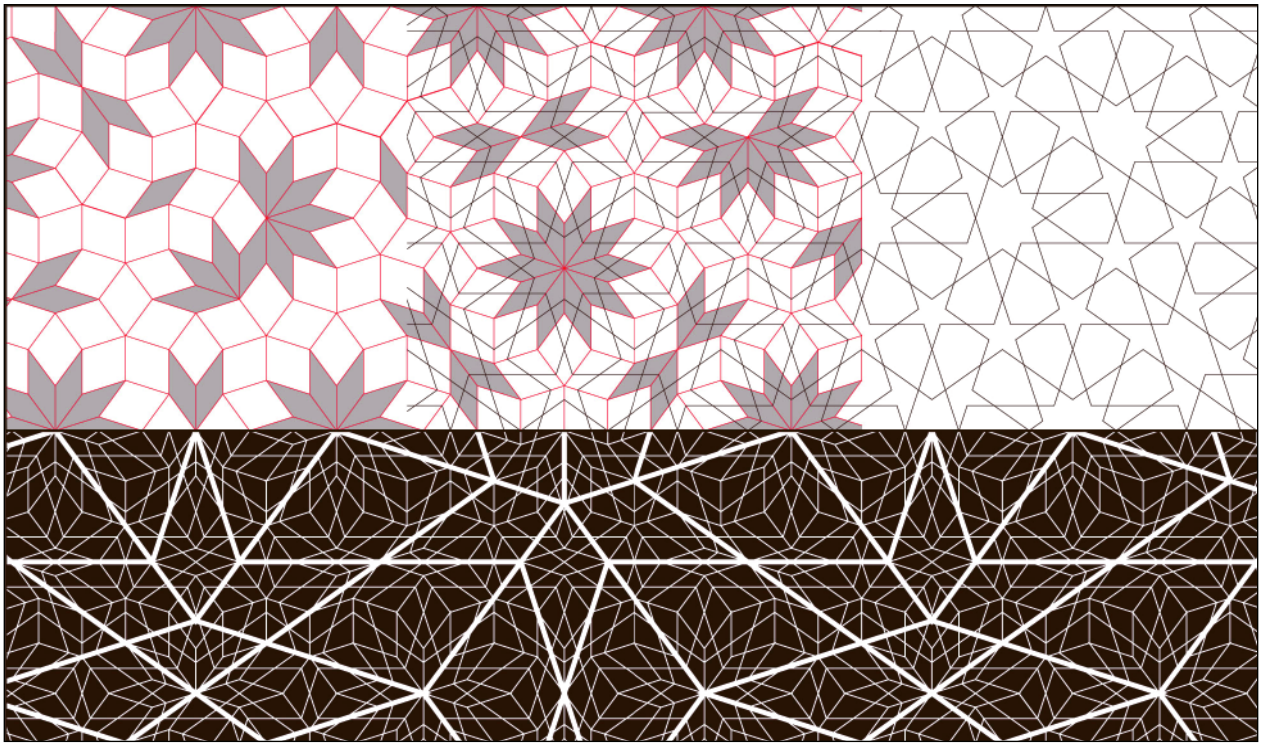

Figure 7. *Top: The Penrose Pattern and the X-Pattern are topologically dual. Bottom: Application of the self-similarity property by a deflation process.*

2. Tiles, Prototiles and “Meta Pattern”

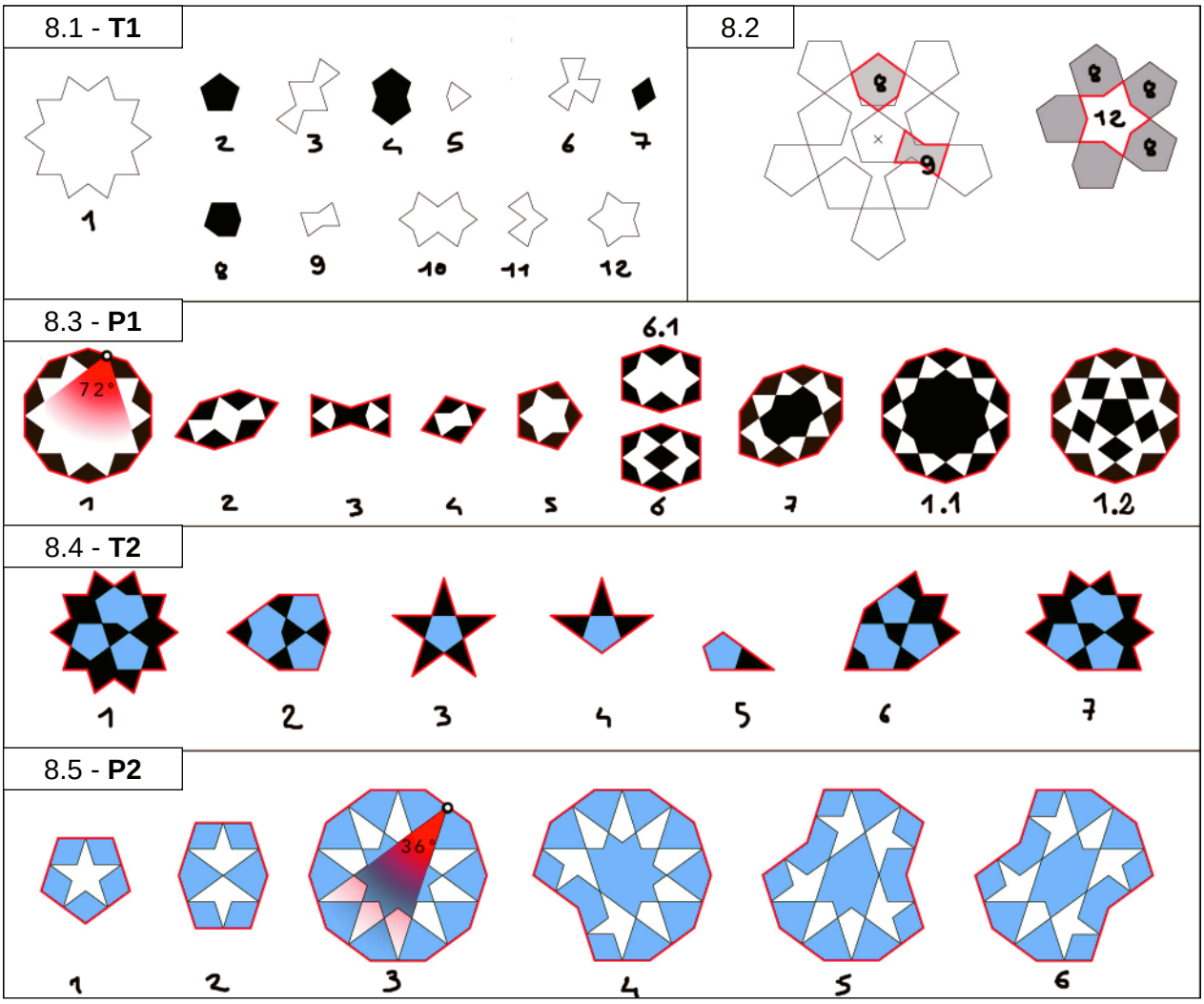

Figure 8.1: *The set of Persian style tiles, T1 .* **2:** *Where the tiles 8,9,12 are coming from.* **3:** *The associated set of prototiles P1 (Number 7, from Cromwell, is not used in this paper).* **4:** *“Meta-tiles” T2, decorated by tiles from T1, and* **5:** *associated set of prototiles P2 (In most cases, we just need 1-4).*

Prototiles, (or “Gerih tiles”, see [4], [6]) are decorated polygons composing an isometric underlying network. The lines drawn from the middle point of each edge of the polygons makes an angle of 72° in P1, 36° in P2. Any pattern made from the set T1 can be seen as made from prototiles (no room enough here for demonstration).

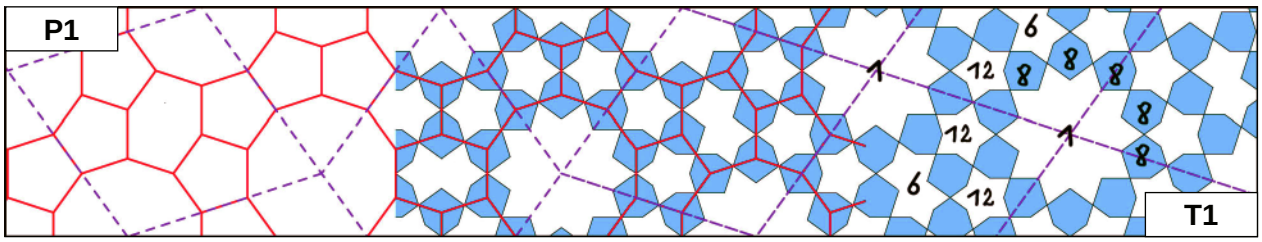

Figure 9: *Example of a pattern using the tiles 1,6,8,12, coming from a variation of V2L1 (fig. 3), and the associated network of prototiles P1. If we substitute P2 to P1, we are getting the fundamental pattern (fig. 1). The same thing occurs if we apply Hankin’s technique (see [5]).*

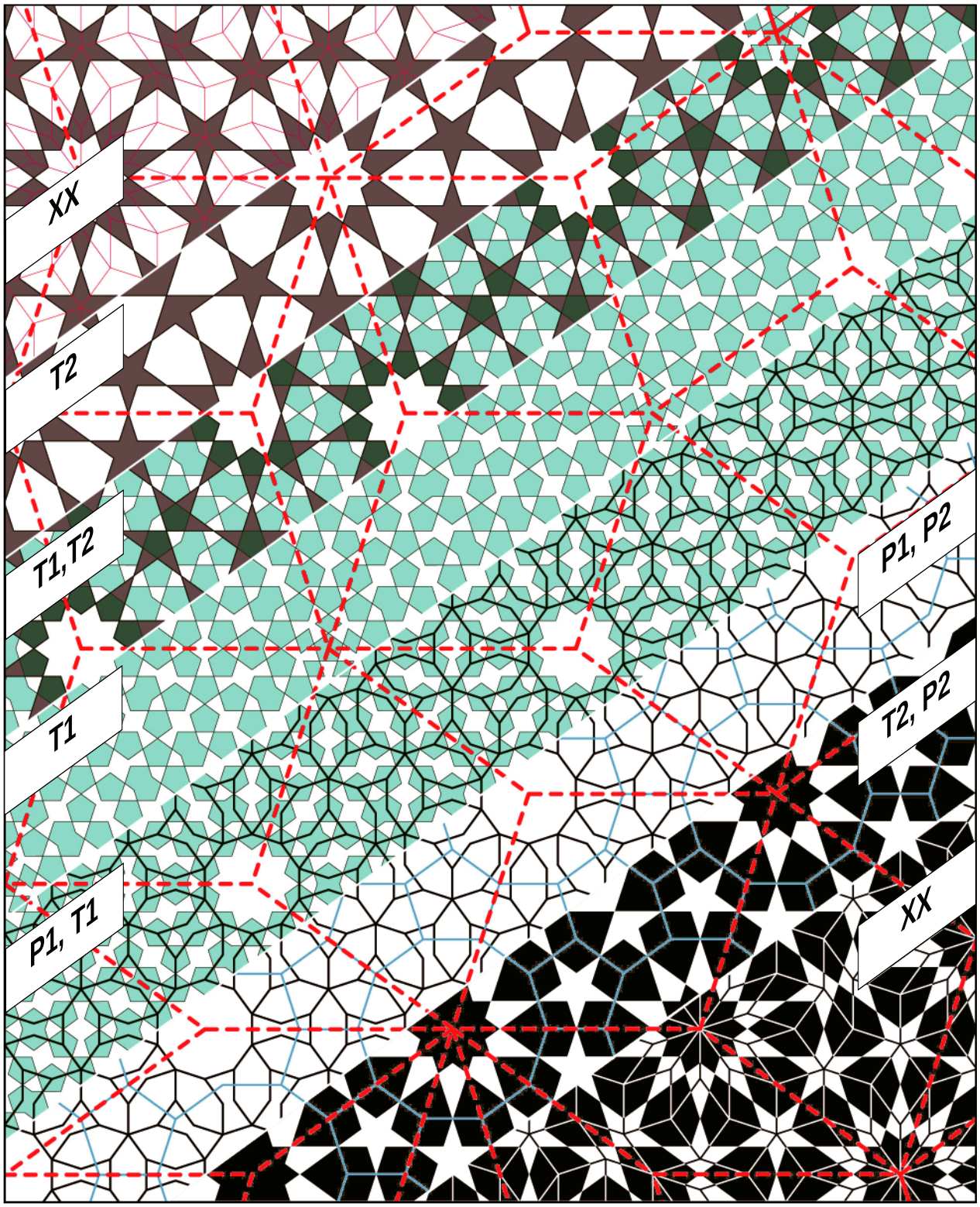

Figure 10: *Variations and relationship from Persian pattern T1, fundamental pattern T2, prototiles P1 and P2, X-puzzle XX and Penrose rhombi structure (Dotted lines). Not full exhaustive.*

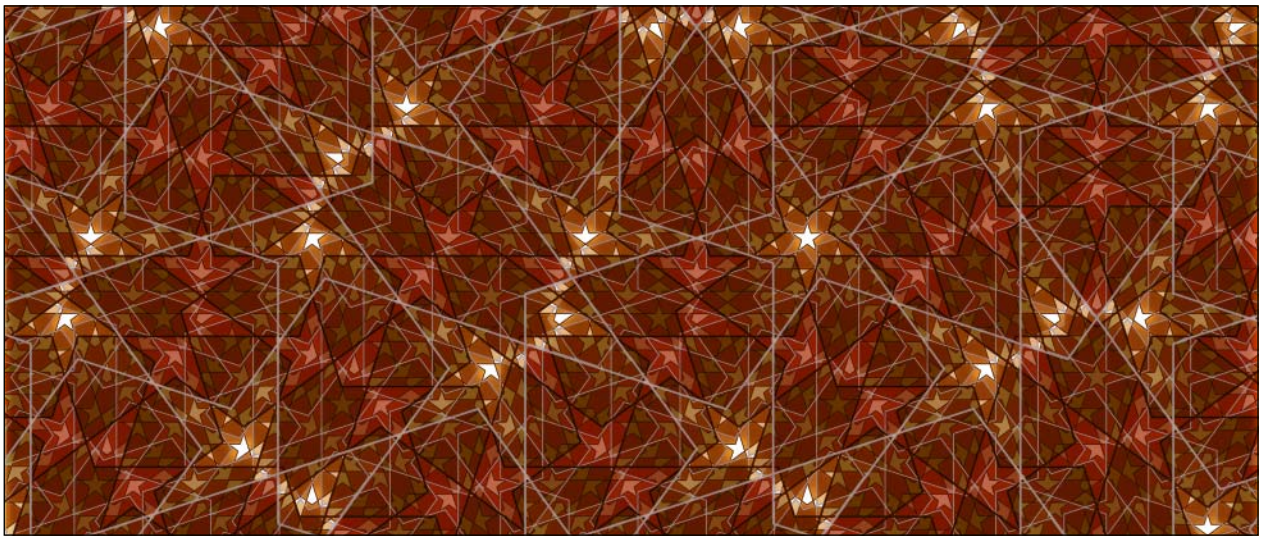

Figure 11: *A free application of those concepts.*

Conclusion

The human mind yearns for a single all encompassing explanation. Hopefully, the world is more rich and complex than any one theory. Concerning the patterns, I have tried here to demonstrate that they cannot be encapsulated by one explanation or point of view. They travel through different theories, styles, countries and ages. Andalousian, Arabic, Persian, the patterns don't care about any border : they are flying.

References

[1] Jay Bonner, *Three traditions of self-similarity in fourteenth and fifteenth century Islamic geometric ornament*, Meeting Alhambra, Proc. ISAMA-Bridges, pp.1-12, 2003.
[2] J.Bourgoin, *LesEléments de l'ArtArabe*, Firmin-Didot,1879. Plates reprinted in *Arabic Geometric Pattern and Design*, Dover Publications,1973.
[3] J.M. Castera. *Arabesques*, Paris, ACR, 1996. English version, 1999.
[4] Peter R. Cromwell, *The Search for Quasi-periodicity in Islamic 5-fold Ornament*, Math. Intelligencer 31 no 1, pp36-56, 2009.
[5] C.S.Kaplan, *Islamic star patterns from polygons in contac*t, Graphics Interface, ACM International Conference Proceeding Series 112, pp.177-186, 2005.
[6] P.J. Lu and P.J. Steinhardt, *Decagonal and quasi-crystalline tilings in medieval Islamic architecture*, Science 315, pp. 1106-1110, 2007.
[7] E.Makovicky, *800-year old pentagonal tiling fromMaragha, Iran, and the new varieties of aperiodic tiling it inspired*, Fivefold Symmetry, ed. I. Hargittai, World Scientific, pp.67-86, 1992.
[8] E.Makovicky, *Another look at the Blue Tomb of Maragha, a site of the first quasicrystaline Islamic pattern*, Symmetry: Culture and Science Vol. 19, Numbers 2-3, pp. 127-151, 2008.
[9] G.Necipogglu, *The TopkapiScroll : Geometry and Ornament in Islamic Architecture*, GettyCenter Publication,1995.
[10] J.Rigby, *A Turkish interlacing pattern and the golden ratio*, Mathematics in School, Vol. 34 N°.1, pp. 16-24, 2005.