A Method for Expandable Regular Tessellation

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Abstract

Due to the rapid changes in the activities of modern society, architecture is in need of adaptable spaces. To meet this need, architects search to find new ways of producing architecture. Movement is accepted as the principle concept by some of designers. They usually reach transformable, deployable or foldable structures with a particular type of mechanism by using advanced mechanical knowledge. There is not a method for the designers who know very little about mechanisms. The main purpose of this study is to develop a method for the construction of expandable planar surfaces. This method has been developed from mathematical regular tessellation technique in the light of architectural, mechanical and mathematical interdisciplinary approach.

Introduction

Today's architecture seeks for the buildings that can adapt to functional, spatial and environmental changes to reach positive impact on the economy and environment. Thus, a new aspect called kinetic architecture has been developed. Architect Santiago Calatrava and mechanical engineer Chuck Hobermann can be accepted as two pioneers. However, many designers deal with a particular type of mechanism for a particular building. There is not a method to construct planar multi loop mechanisms which are appropriate for adaptable building facade. This investigation focuses on the development of a method for the construction of tiles and links that make up an expandable regular tessellation. In this paper, it is intend to propose an expandable surface with only single internal mobility, whose size can be changed easily for different purposes.

Duality of the Tessellation

The dual of tessellation is formed by joining the center of each polygon as a vertex and joining the centers of all neighboring polygons. The number of sides remains the same. Moreover, Grünbaum and Shephard describe the dual tessellation on their book as, "two tilings are said to be dually situated to each other if they lie in the same plane, every vertex of one is an interior point of a tile of the other, every tile of one contains precisely one vertex of the other, and crosses, just one edge of the other" [1].

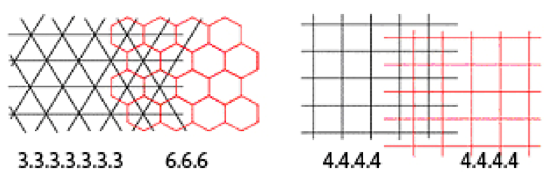

Figure 1: *Dual of regular tessellations.*

Also, they claim that, tessellation and its dual are homomorphic to one of the two dually situated tessellations. Thus, any motion or change of scale affects each other [1]. The triangular and hexagonal tessellations are duals of each other, while the square tessellation is its own dual (Figure 1).

Motions on Planar Surfaces

In plane or two dimensional Euclidean spaces there are three types of motions, as pure rotation, pure translation and complex motion (Figure 2). The individual bodies that make up a mechanism are called the members or the links. From the mechanical point of view, a link is assumed to be completely rigid. The links in a mechanism are connected by pairs. The connection between two links is called a joint or a kinematic pair [2].The method described here uses only revolute joints. Revolute (R) joints (Figure 3) permit two connected elements to rotate with respect to each other around a rotation axis.

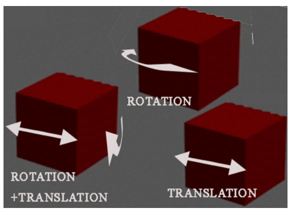

Figure 2: *Motions on the plane.*

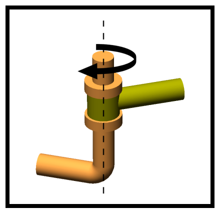

Figure 3: *Revolute joint.*

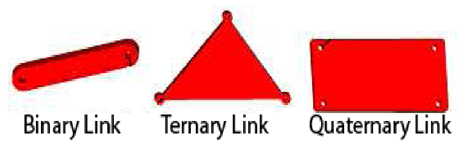

Figure 4: *Links of different order.*

A kinematic chain is an assembly of links that are connected by joints. Each link in a valid kinematic chain should have at least two distinct joints. As an example, links with two pair elements are called binary link, three pair elements are called ternary link and so on (Figure 4).

Method for Kinetic Regular Tessellation

This investigation tries to find an answer to the question of how an expandable regular tessellation can be constructed by using regular links. The most important point in this case is that the planar mechanism reaches the regular tessellation at the beginning and the end of the motion. This method consists of three steps successively building upon each other.

1- To decide the first link's type and size among the regular tessellations.
2- To determine the second link's type and size.
3- To determine the construction order of the links.

Expandable Hexagonal Tessellation

Step 1- By selecting the hexagonal tessellation $\{6^3\}$ as demonstrated in figure 5, the type of the first link is determined. It is hexagonal link as shown figure 6. The size depends on the designer's decision.

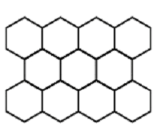

Figure 5: *Hexagonal tessellation $\{6^3\}$.*

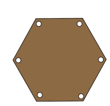

Figure 6: *First link (Hexagonal).*

Step 2- In the second step, a hexagonal tessellation is drawn according to first link size. Then, the type and the size of the second link are determined by pointing one vertex to the three neighboring vertices as shown in figure 7.

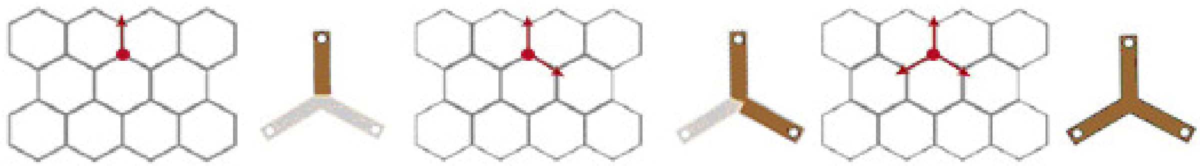

Figure 7: *Process of obtaining the second link (Ternary).*

Step 3- To determine the placement of the links, blue colored hexagonal tessellation and its red dual are drawn as shown in figure 8. Center points of the six hexagonal links are placed on the intersection points of the hexagonal tessellation and the dual. One more hexagonal link is placed on the center point of the hexagon. Then the six ternary links are placed on the vertices of the blue hexagon. Six more ternary links are placed in between the hexagonal links.

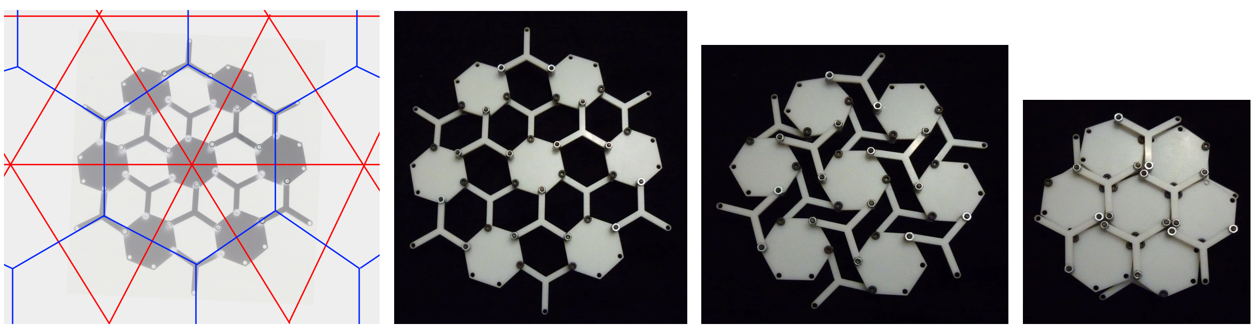

Figure 8: *Placement of the expandable hexagonal tessellation and its expansion.*

This method can be applied to the other regular tessellations. The triangle and hexagonal tessellations are dual of each other. The construction of the triangular tessellation is the same with the hexagonal one. In this case, the first link is the ternary and the second link is the hexagonal.

Expandable Square Tessellation

The third form of the regular tessellation is the square tessellation. It is possible to construct the expandable square tessellation with the same method.

Step 1- By selecting the square tessellation $\{4^4\}$, type of the first link is determined. It is a quaternary link and has a square form. The size depends on the designer's decision.
Step 2- In the second step, a square tessellation is drawn according to the first link size. Then, the type and the size of the second link are determined by pointing one vertex to the four neighboring vertices. It is a plus form quaternary link as shown in figure 9.

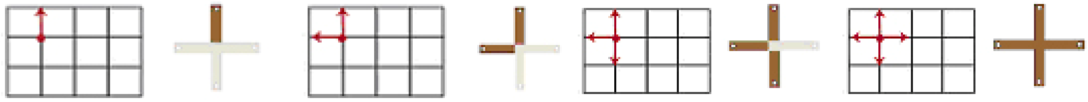

Figure 9: *Process of obtaining the form of the second link.*

Step3- To determine the placement of the links, square tessellation and its dual are drawn as shown in figure 10. Four square form links are placed on the points where the square tessellation and its dual intersect. One more link is placed on the center point of the square. Then the plus form links are placed in between the square form links.

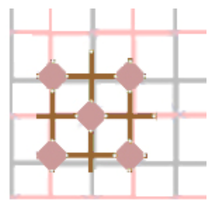

Figure 10: *Placement of the expandable square tessellation.*

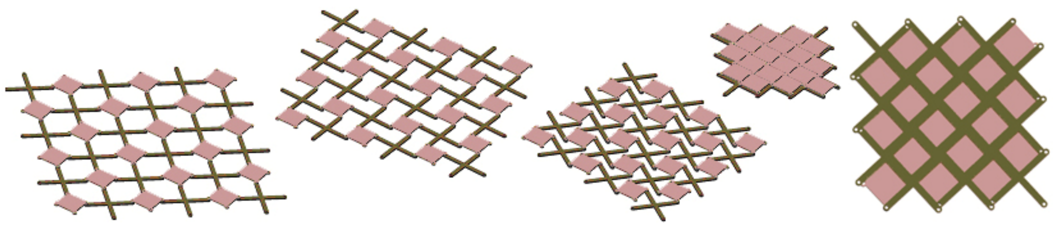

Figure 11: *Open and closed forms of the expandable square tessellation.*

Conclusion

This study tries to develop a method which is concerned with the construction of tiles and links that make up an expandable regular tessellation. The expansion process is achieved by the internal single degree of mobility, which makes the regular tessellations expand repeatedly. The size of the links can be determined according to the application. In the future, this investigation aims to apply this method to the existing eight semi regular tessellations.

At the same time, this study tries to highlights the importance of the relationship between mathematical knowledge, mechanism science and architecture in kinetic architecture field.

References

[1] Grünbaum, B. and Shephard, G.C, *Tilings and Patterns*, W. H. Freeman and Company, New York, 1986.

[2] Norton, Robert. L. *Design of Machinery: An Introduction to the Synthesis and Analysis of Mechanism and Machines,* McGraw-Hill, Third Edition, Singapore, 2004.