# Mandelbrot Sound Map - A Tool for Mapping Fractals into Sounds

Boris Kerkez
Department of Mathematics and Computer Science
Ashland University
Ashland, OH, 44805, USA
E-mail: bkerkez@ashland.edu

## Abstract

Fractal images enjoy a great deal of popularity due to their aesthetically pleasing qualities. While the generation of fractal images is a topic that has received a lot of attention in the past, only a few attempts have been made to create musical compositions using fractals and their audible properties. This paper describes one possible process of mapping the elements from the Mandelbrot Set fractal to audible musical notes. The *Mandelbrot Sound Map* software tool has been created to implement the mapping of set elements to audible outputs. This software produces audible compositions based on a parameterized traversal of a specific portion of the Complex number plane containing the Mandelbrot Set, and creates musical files that can be listened off-line as well.

## Introduction

Ever since the early days of computer-generated images, fractal graphics has enjoyed immense popularity due to its pleasing symmetrical aesthetic nature and relative ease with which fractal images can be generated. Fractal images can be generated by passing each point of the Complex number plane through an iterative procedure and coloring the pixel corresponding to a specific Complex number based on some established criterion. One of the most recognizable fractals is the Mandelbrot Set [3] (see Figure 1). While the Mandelbrot Set is self-similar at magnified scales, the small scale details are not quite identical to the whole. In fact, the Mandelbrot Set is infinitely complex. Yet the process of generating the set is based on a very simple equation involving Complex numbers. A graphical interpretation of the Mandelbrot Set can be generated by passing a Complex number $C$ through the iteration $z_{i+1} = z_i^2 + C$, where $i \in Z^+$ and $z_0$ starts at zero. For a Complex point to be in the Mandelbrot Set, this iteration will produce Complex points whose magnitude stays bounded by 2. Points that diverge, however, are of most interest in graphical representation, as the speed of their divergence can be measured and used to modulate the actual color of the point displayed.

*Mandelbrot Sound Map* is a tool that follows a process similar to the one employed in graphical representations to map points in the Complex number plane (outside of the Mandelbrot Set) to audible musical notes. The actual mapping process, as well as the implementation of this process, is described in the next section. The Future work section describes ideas for additional maps, including mapping Complex points onto multiple tracks with different instruments and characteristics.

## Mapping the Mandelbrot Set onto Notes

The idea of mapping mathematical constructs into audible events is not a novel one. Indeed, the very concept of a musical scale is based on a precise mathematical progression of frequencies that result in harmonic progression with qualities that are pleasing to the human ear. More recently, researchers have established methods of mapping prime number sequences into sounds [4] and methods of musical composition based on mathematical processes, as in [2] and [5]. The approach discussed here is much more similar to the maps of prime number sequences onto sounds, rather than to the approach that utilizes variations from a chaotic mapping described in [1] and [6].

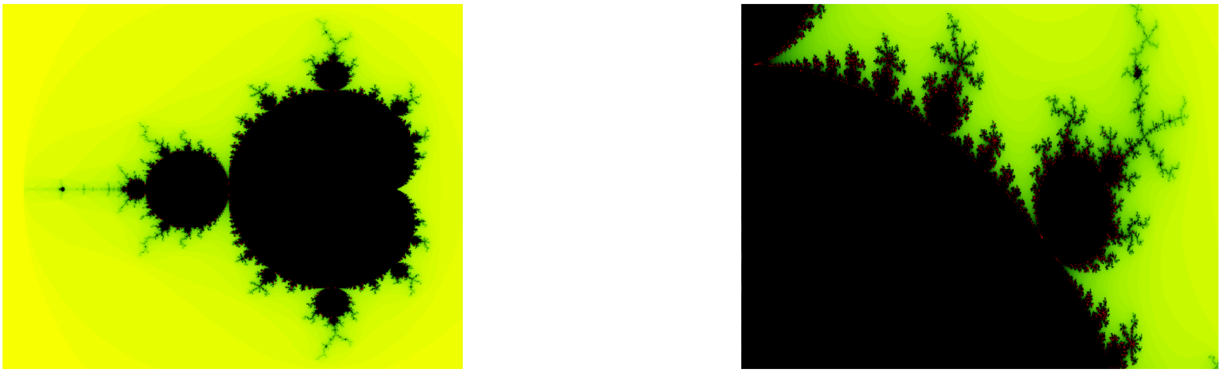

**Figure 1**: *The Mandelbrot Set, as produced by the application. Left: The set at full zoom (0-100%). Right: The set with a non-uniform zoom of 66%-84% on x-axis, and 66%-78% on y-axis (see Figure 2)*

The process of mapping points of the Complex number plane onto musical notes described here is based on the established process that generates graphical representation of the Mandelbrot Set. Just as Complex numbers that belong to the Mandelbrot Set are not very interesting with respect to their graphical representation and are colored uniformly (usually black), these particular numbers produce no sounds and they are ignored in the mapping process. As for points outside the Mandelbrot Set, the speed of their divergence (i.e., their *escape velocity*) is measured and points are mapped onto different notes based on their escape velocities. The range for these escape velocities is from one up to the number of iterations. The escape velocities are then mapped onto the set of all possible notes $S$, which is determined based on the current scale selection, as well as the root key and the number of octaves selected. Given a Complex point $C$ that lies outside the Mandelbrot Set, and a set of all possible notes $S$, the points' escape velocity is divided using the modulus operation by the cardinality of $S$. The resulting number $n$ will be between one and $|S|$; the sound produced by the Complex point $C$ is the $n$-th note in the set $S$.

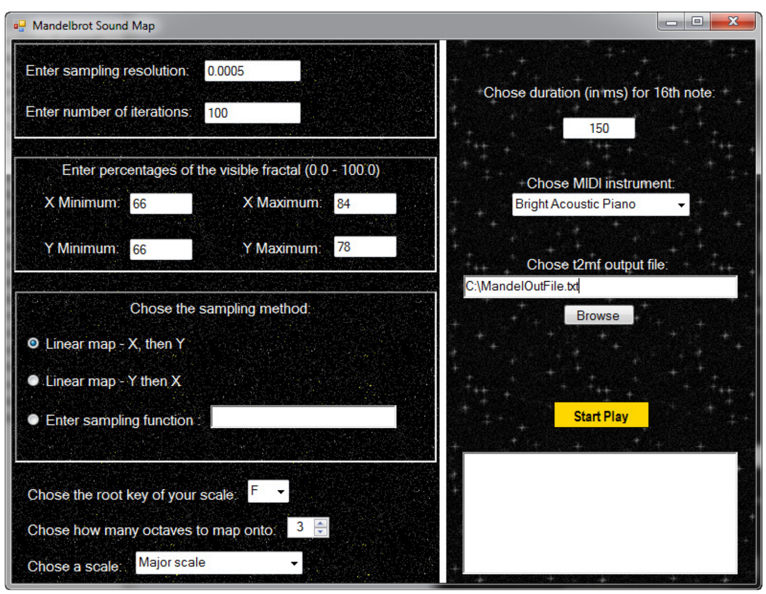


**Figure 2**: *The Mandelbrot Sound Map Application - Main window*

Figure 2 shows the *Mandelbrot Sound Map* main application window. The application was written in Microsoft Visual C++ and is available for the Windows platform (the tool can be downloaded from *http://personal.ashland.edu/bkerkez/MandelbrotSoundMap*). The main window allows the user to specify a variety of parameters that dictate the exact process of mapping Complex points onto sounds.

**Sampling method.** Another difficulty with mapping fractals onto sounds is that unlike images, musical constructs have a temporal element associated with them. While all points of the fractal image can be computed and displayed at the same time, sounding all notes at once, although theoretically possible, is not a practical option. When producing musical constructs, there clearly needs to exist a temporal element of the mapping process. For the purposes of this application, notes that appear in sequences are converted to single notes, with their duration being proportional to the length of the sequence. The traversal of the Complex plane can be either by rows or by columns (row-major or column-major order) of the selected portion of the fractal. In the future, users will have the ability to specify their own traversal function and thus affect the temporal sequence of sounds produced.

**Sampling resolution.** Since any subset of the Complex number plane contains an infinite number of points, for all practical purposes, we need to make the Complex plane discrete by specifying the resolution at which to sample subsequent Complex points. This is the distance, on both x and y-axis, between two adjacent points in the Complex number plane considered by the mapping process. The finer the resolution, the more Complex points are considered in the mapping process. Too small of a resolution, however, can compromise the performance of the application, as there will be a large number of points to consider.

**Percentages of the visible fractal.** Describes the non-uniform scaling factor of the Mandelbrot Set with ranges from 0% to 100% on both axes. The full scale factor, from 0% to 100% on both axes, is shown on the left in Figure 1. The right part of Figure 1 shows the rendering of a non-uniform scale factor of 66%-84% on the x-axis, and 66%-78% on the y-axis.

**Other mapping parameters.** The *Mandelbrot Sound Map* allows the user to specify additional parameters, such as the musical scale, its root note, and the number of octaves to map onto. Users can also specify the duration of individual notes, as well as chose a General MIDI instrument that will play produced sounds.

**Figure 3**: *The beginning of the composition produced by the tool with parameters shown in Figure 2.*

An example of a composition produced with the *Mandelbrot Sound Map* tool is shown in Figure 3. This figure shows the first 28 bars of the composition produced by the tool. The map that produced the score shown in Figure 3 utilized the F-major scale and mapped points onto three octaves with a simple modulus division method, and with all the other parameters shown in Figure 2. Figure 4 shows the same composition where notes are represented in a MIDI editor. There is a definite visual pattern to the notes visible in Figure 4, where three chunks of subsequent notes show a downward slope and tendency. The *Mandelbrot Sound Map* tool also creates a file for the "t2mf" command-based, free software tool, which takes a text-based specification of MIDI events and transforms them into a standard MIDI file. This way, compositions played with the help of the *Mandelbrot Sound Map* tool can be exported, converted, and played with many applications that support the General MIDI file format.

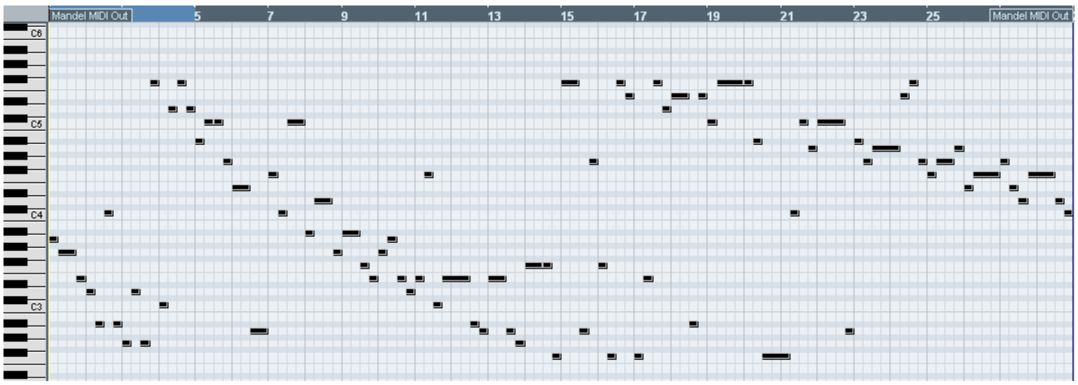


**Figure 4**: *The score from Figure 3 shown in a MIDI Editor.*

## Future Work

While this paper describes one possible mapping of the Mandelbrot Set into sounds, there are many other possibilities that can be explored further. In particular, traversing the Complex plane by a user-specified linear or non-linear function has a potential to produce interesting sound sequences. Many musical compositions consist of multiple tracks, and such effect can be implemented by initiating more than one simultaneous traversal of the Complex plane within a specified temporal structure. It would also be very interesting to map Complex points onto drum sounds, to investigate if there is a periodic pattern in produced musical sequences that can be considered rhythmical in nature. In addition to the Mandelbrot Set, there are many other famous graphical representations of other fractals, and mapping of these fractals onto musical constructs will be explored in future research.

## References

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