Build Something Beautiful and Interesting with Giant Triangles: An Entry to Deep Mathematics

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Abstract

The Giant Triangles are brightly colored lightweight, 1 meter edge length, equilateral triangles, which can quickly be assembled and reassembled into polyhedra. This workshop activity was developed in an urban teacher preparation program for students with little or no prior exposure to conceptual or investigative learning approaches. It involves groups building their own creations with the triangles which are to be 'beautiful and interesting' and then explaining why their creation is beautiful and interesting. Taking each creation in turn, we will discuss how to bring out the inherent mathematics and identify key teaching opportunities in whole group discussions.

Introduction

Several Bridges workshops have already been carried out using the giant triangles e.g., [1], [2], [3]. However, this workshop will introduce an activity devised by M. Connell to address the needs of his pre-service teacher candidates at University of Houston-Downtown. Many have never experienced valuable investigative or conceptual approaches to learning mathematics. Rather, they expect mathematics to be restricted to fact and skill based practice. Thus, as an entry to deep mathematics, they need a different approach, which at the outset does not invoke their restrictive preconceptions of math. This is achieved by providing open ended instruction, with no explicit math fact or skill goal, but which nonetheless gets them doing deep mathematical thinking. The educational theoretical framework used for this workshop (not covered in [1], [2] or [3]) is that of *Actions on Objects* [6] whereby there is mathematics inherent in a manipulative that must emerge from its use.

Workshop Format

Part 1: Building in groups and sharing. Participants will break into small groups and create their own shapes with the triangles that they think are 'interesting and beautiful', without having to conform to any mathematical preconception such as being regular polygons or polyhedra. Shapes do not need to close up and can have more than two triangles to an edge. Shapes can also be placed on top of other shapes. Groups will reflect both on the process by which they generated their shape or shapes and also the end result. What was beautiful and what was interesting, what was tried and what worked, were there any surprises? Again, these reflections do not have to be in terms of formal mathematical concepts or terminology. Finally, there is a show-and-tell for each group's shapes and reflections to the others in the workshop.

Part 2: Discussion of formal mathematics teaching opportunities and connections to art. Each group will spend a few minutes discussing what formal mathematics can be illustrated in the shapes, and how this gives rise to teaching and learning opportunities [6]. Then, the whole group will review all the ideas and reflections that people wish to share as everyone considers and compares all the shapes. Contributions from art teachers or teachers from other disciplines will be welcome in addition to relating the workshop to the NCTM process strands [5] and how the process integrates the van Hiele levels of geometric thought [4], [1].

Part 3: Continued shape building and review. The remainder of the workshop will be spent on further building. Groups who want guidance will be offered suggestions including: How many ways can the colors for faces of a given shape be chosen to give a different symmetry pattern to the colored figure? How many shapes can be made with a fixed number of triangles? The review will catalogue the mathematical concepts that participants refer to, perhaps informally, in their explanations of their shapes. These give opportunities for introduction of formal concepts and discussions of definitions. Specifically this will extend some concepts from 2D to 3D, such as: parallel lines extending to coplanar and parallel faces, angles between lines, acute, right, obtuse, and reflex extending to dihedral angles. Comparing convexity and concavity in 2D and 3D can lead to a discussion of definitions rather than just appearance. Also when considering reflection and rotation in 2D and 3D, the vocabulary and definitions used for the 2D cases sometimes need re-examining for 3D, such as 'point symmetry' or 'line symmetry'.

Conclusion

Participants will experience mathematical exploration through the shape building activity. Then, participants will develop pedagogical ideas and practices through whole group discussion to deepen the mathematical activities, concepts and language.

The Authors

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References

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