The Rhythm of a Pattern

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Abstract

This paper explores the relationship between music and visual art. By implementing aspects from fractal geometry, aperiodic tilings and traditional geometry a new method is revealed where rhythm is translated into pattern allowing artworks to be read as musical scores and music to be represented in geometric form. The method explained here is the foundation of the project 'A Hidden Order' by Sama Mara and composer Lee Westwood.

Introduction

"There is geometry in the humming of the strings, there is music in the spacing of the spheres"
Pythagoras [1]

The relationship between music and visual art is a topic which reaches back thousands of years, and recurs repeatedly in the arts and sciences. The artist Wassily Kandinsky wrote "The sound of colors is so definite that it would be hard to find anyone who would express bright yellow with bass notes, or dark lake with the treble,"[2] exemplifying the intuitive understanding of this relationship that we all share.

There are fascinating physical relationships between sound and form seen in experiments such as those in Cymatics, where liquids or particles are exposed to the vibrations of sound waves and take on particular forms related to the frequency of the sound; and the harmonograph, which creates patterns relating to specific musical harmonies created by the use of pendulums swinging in relative frequencies analogous to the musical ratios. There is a strong tradition of visualising music within the Western arts, as seen in the works of the abstract painters of the 1900's such as Mondrian, Kandinsky and Klee, as well as animated pieces by the likes of Oskar Fischinger and John Whitney.

Sonification of art is perhaps less explored, but the use of the Golden ratio through the implementation of the Fibonacci series in music is one place where we find the influence of geometry and the visual arts on music as seen in compositions by Béla Bartók [3] and Debussy [4]. There are also many examples of composers relating pitch or key centres to colour such as Scriabin [5] and Messiaen [6].

Newton, on proving the spectral nature of light, related the seven notes of the musical scale to seven colours of the spectrum in his book Opticks. We also have contemporary theoreticians exploring the relationship, such as the recent celebrated work of Dmitri Tymoczko, 'A Geometry of Music', which views Western musical tonality through geometric space.

The following paragraphs document a new method that allows for music to be visualised as unique patterns, and for a certain family of geometric artworks to be read as musical notation. Through this process, rhythm is translated into pattern and pitch relates to colour. The pitch / colour aspect of the method described here is not new and shall not be expounded in depth.

Method

Colour & Pitch. The relationship between colour and pitch is relatively simple when we consider sound and light as waves. The amplitude of the wave relates the brightness of colour to the loudness of sound, whilst frequency relates hue to pitch. Lastly, the complexity of the wave form relates saturation to timbre. This creates a continuous relationship between colour and sound that is not dependent on any musical system.

The hue-pitch relationship requires further attention. An intuitive relationship often implemented is that one octave of sound relates to the entire visible spectrum plus the magentas. But this has an inherent issue in that humans are capable of hearing up to 10 octaves (doubling of frequency from 20Hz, 40Hz 80Hz... up to around 20,000Hz) whereas we can see just under one doubling of frequency of light waves (390 to 700nm). So to differentiate between the range of octaves as the pitch goes higher through the octaves the colour becomes lighter, and as we descend through the octaves the colour becomes darker. But this lighter / darker aspect of colour is already related to the loudness of the note via the amplitude of the waveform. This means that the same colour may be produced by a low quiet note or a higher loud note.

Rhythm & Pattern. To reveal a mapping between pattern and rhythm, we explore what happens if we assign a square to represent one unit of time. This unit of time shall be referred to as a beat.

How would two beats be represented as form? We could at this point place another square beside the first. However we shall explore what happens when we increase the area of the square by a factor of 2 (see Figure 1). We can repeat this process of doubling the area to create a square 4 times the original area, then 8 times, and so on. We shall refer to each nesting square as a 'generation-square' so the first square is the 'first generation-square', the square of area two as the 'second generation-square', area 4 as 'third generation-square' and so on.

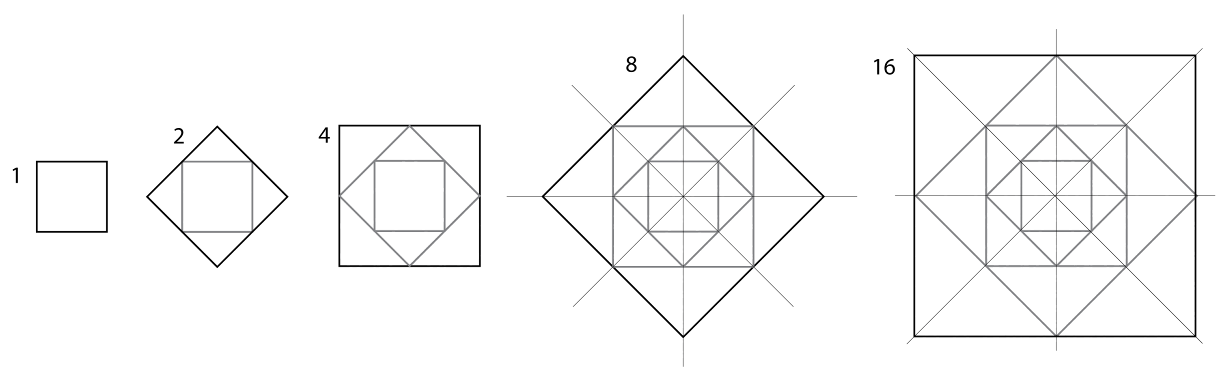


Figure 1: *Progression from one square representing one beat, to double the area representing double the length of time, and the process repeated three more times to reach sixteen times the original area.*

At this point we need to find the placement of the individual beats within this form. We already have the positions of beat 1 and beat 2, and know that beats 3 and 4 must lie within the third-generation square beats 5,6,7 and 8 within the fourth-generation square and beats 9 to 16 are within the fifth-generation square. This series of nested squares naturally divides space into a grid of isosceles triangles. We imagine tracing a path through one eighth of these triangles to mark out a sequence of beats as shown in Figure 2.

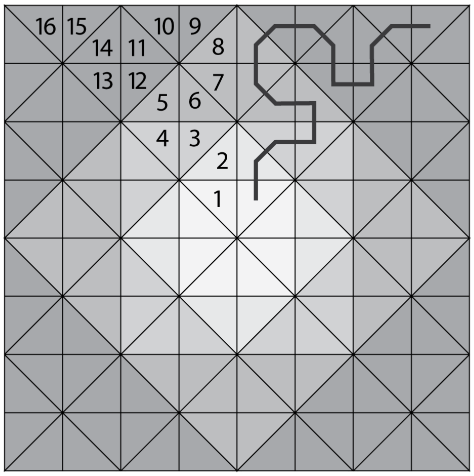


Figure 2: *Sequence of cells each representing a beat. The top right area of the grid shows a section of a Sierpinski space filling curve.*

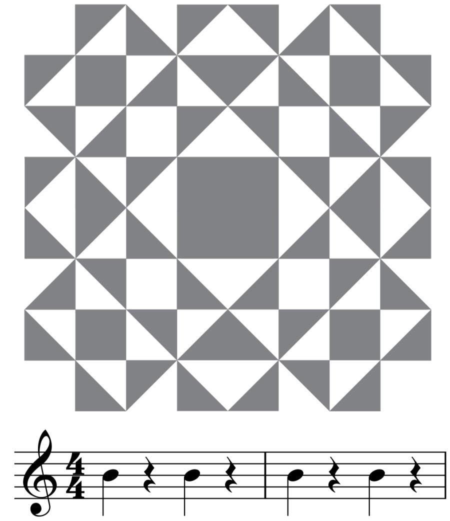


Figure 3: *Simple rhythm and related pattern after 16 beats. Dark areas relate to beats played.*

Only one eighth of the grid needs to be created. This is then reflected/rotated around to complete the radial symmetry of the grid. The cell order of this grid follows the principles of a Sierpinski space filling curve, which may be created by joining the centers of each cell in order with a continuous line, see Figure [2].

We now have a 1:1 mapping between time and space, where each unit of time is related to a unique cell placement in the grid, and where length in time corresponds to area in space. If we mark the cells with an accented beat to be dark and the rests to remain white, we may visualise a particular rhythm as a unique pattern, and likewise create a pattern and read it as a rhythmical motif.

Alex Mclean and Tim Blackwell explored this same principle of visualising music using a space filling curve and their experimentations can be seen on the web page entitled 'Peano Curve Weaves of Whole Songs' [7]. The results were significantly different as the pulse of the music was not synced with the curve/grid so that a given cell of the grid was not related to a given beat of the music.

When visualising rhythms with the method described above a problem becomes evident in that simple rhythms do not necessarily relate to simple patterns. For example one of the most basic rhythms, where every other beat is sounded, leaving the others as rests, creates a comparatively complex pattern (see Figure 3). Using this system it appears that, to create simple patterns, the corresponding rhythms must be palindromic, which is not a standard way of approaching rhythm in music.

We have been dealing with rhythm and pattern on a purely mathematical basis and haven't yet drawn upon the human experience of the two. When considering the qualitative aspects there are basic relationships which ideally would hold true. Two of these are that the complexity of a rhythm should be reflected in the visual counterpart, so a simple rhythm has a simple pattern, and a sparse rhythm should create a sparse pattern, and they should increase in density together. An approach to meeting these requirements would be to embed the palindromic structure within the cell order itself as follows:

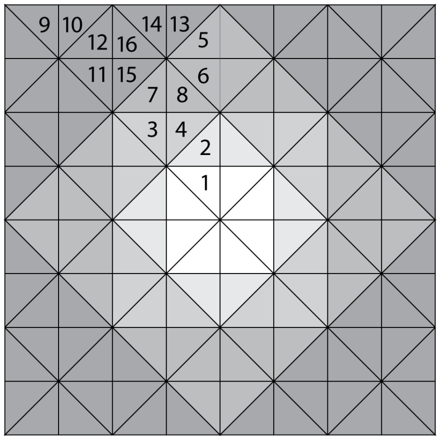


Figure 4: *Alternative cell order*

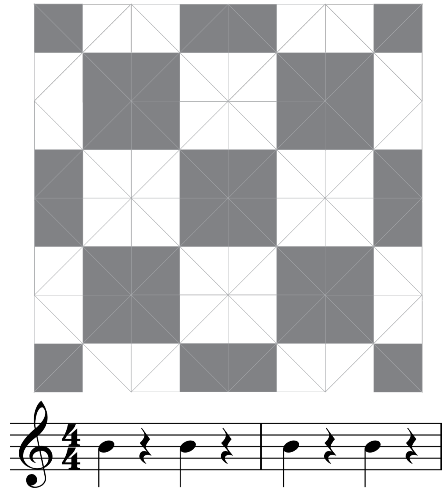


Figure 5: *Simple rhythm and related pattern after 16 beats with the new cell order. The chess board design feels intuitively more appropriate as a visual representation of the rhythm.*

With this new cell order the contiguous nature of the space filling curve is lost. However the result works much more effectively, where simple rhythms create simple patterns and the complexity of both increase with each other (see Figure 5).

The sequence is created through a series of reflections, where each reflection line runs along the edge of a generation-square, naturally doubling the cells in the grid, and resulting in the first cell in each generation placed at the vertex of the new generation-square, see Figure 6.

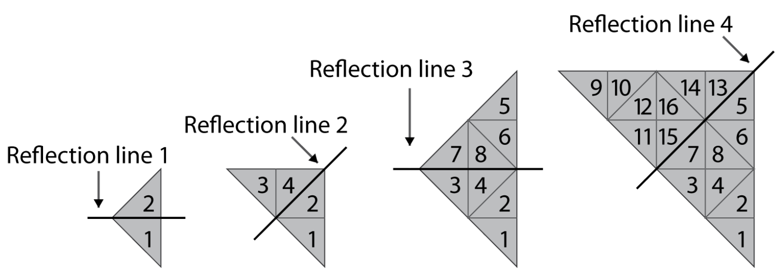
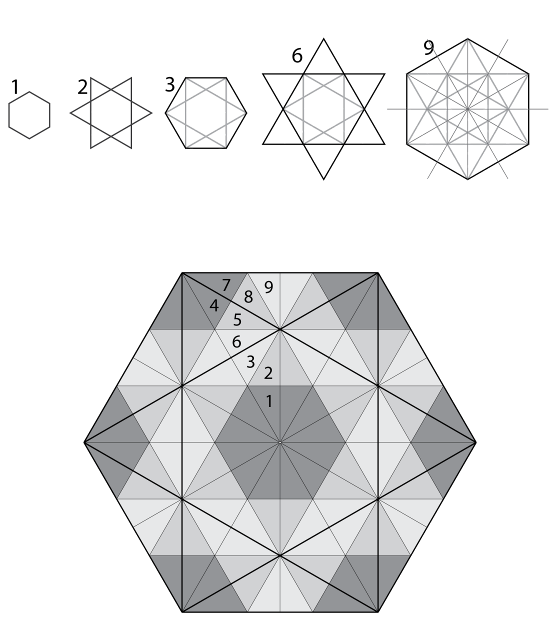


Figure 6: *Construction of a section of the grid with new cell order. Each new generation is created by a reflection about a line lying on one edge of a generation-square. Every cell in the grid can be located through a unique combination of reflections.*

Hexagon and 12/8

The same process as with the square may be extended to the hexagon. Starting with an initial hexagon the area is doubled making a hexagram, the next step triples the original area and resolves to a hexagon thus completing a generation. This may be repeated indefinitely resulting in a grid based on the tripling sequence 3,9,27... See Figure 7. The cell order is also created using reflections as in the square grid. The hexagon may also resolve to the next generation after three steps so quadrupling the area at each generation, with the sequence 4, 16, 64... see Figure 8.



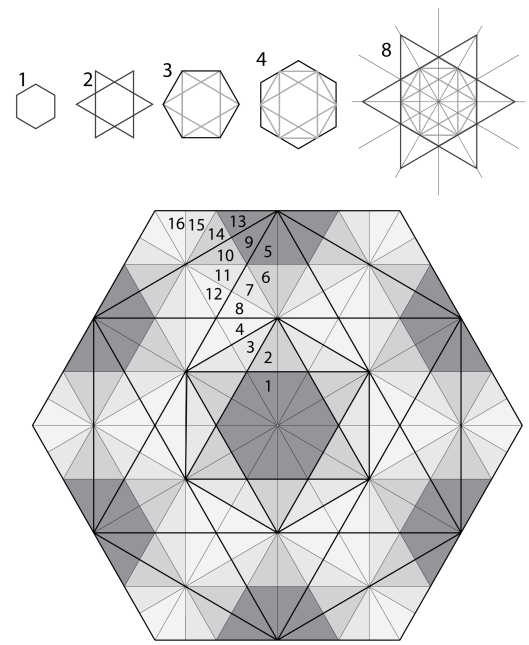
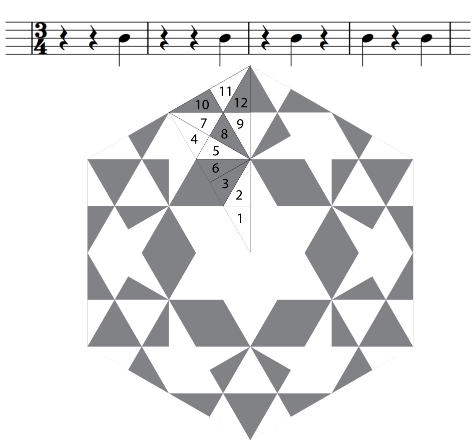


Figure 7: *The doubling of the area of a hexagon results in a hexagram, and the hexagon is resolved at the next step with a tripling of the area of the original hexagon, providing a generation based on a factor of three. The resulting grid and cell order are displayed up to 9 beats.*

Figure 8: *The hexagon may also be resolved with a quadrupling of the original area.The first steps up to eight times the area are displayed here, with the resulting grid and cell order up to 16 beats.*

The fact that the hexagon resolves with an increase of area three times or four times the original area, means that we can create a hexagon grid which resolves after groups of 3 beats, or 4 beats, or any combinations of these e.g. 9, 12, 16, 18, 36 etc... This opens up a lot of possibilities of exploring different note groupings, time signatures and accordingly, musical styles. The method enables us to visualize rhythms as patterns and read a certain family of patterns as rhythm. Figure 9 shows a traditional Flamenco compás visualised as a pattern, and Figure 10 transcribes a classic Islamic pattern into musical notation.



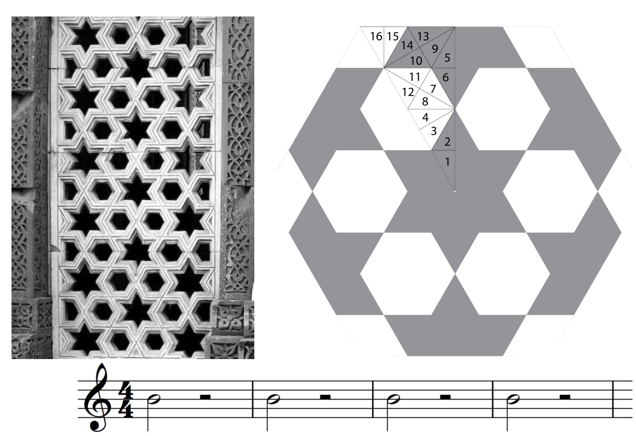


Figure 9: *Visualisation of a flamenco compás*

Figure 10: *A simple classic pattern transcribed into music. Photo By Michael Hoy, via Wikimedia Commons*

Time Signature and Grid Symmetry

The symmetry of the grid relates directly to the time signature in music. The growth of the area between subsequent generations of the process determines the number of beats in a bar. For example a square resolves to a larger square with a doubling of the area, so the square grid relates to a 2-beat bar, but may also relate to a 4-beat bar, 8-beat bar or any number in this doubling sequence. The hexagon resolves to a larger hexagon with a tripling of the area, relating to a 3-beat bar, or 4 times the original area, so relating to a 4-beat bar – see above.

Aperiodic Rhythms

The use of pentagonal and octagonal tilings introduces an added dimension to the rhythmical counterpart. To explore these symmetries we draw upon the work that has been done in recent decades in aperiodic tilings by using the classic example of the Penrose Tiling composed of Robinson Triangles to illustrate a mapping into rhythm.

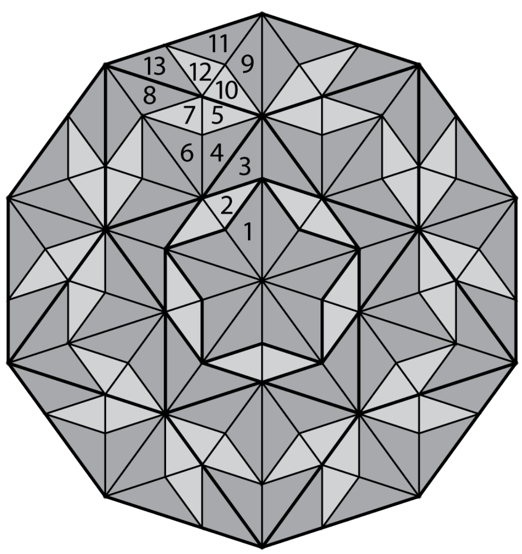


Figure 11: *Penrose tiling with cell sequence*

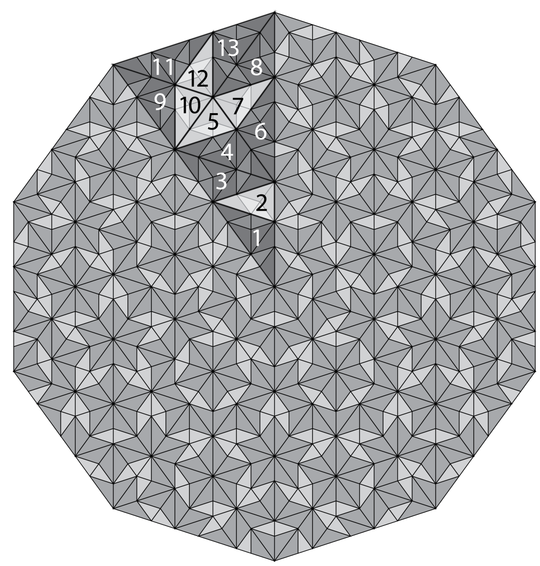


Figure 12: *Penrose Tiling bar sequence, with bar lengths of 5(light) and 8 beats(dark)*

The sequence of beats will never settle into a periodic repeat but will constantly be evolving. The bar lengths may be chosen at different sizes by grouping different numbers of cells, but they will always be two consecutive Fibonacci numbers. Bar lengths of 5 and 8 are illustrated in Figure 12.

The two cell types, although different in size, are treated as equal lengths of time. This seems acceptable given that they are both projections from the same higher dimensional cubic structure [8]. However this raises the concern that when the grid is developed the two sections which are the same shape as these two forms (expressed as bars) contain different amount of beats, and so correspond to different lengths of time.

Example Artworks from the A Hidden Order Collection

Bringing together the colour/pitch relationship and the rhythm/pattern described in this paper it is possible to create artworks from music. Whole pieces of music may be translated into form either by continually expanding outwards from the centre (see Figure 13, 14 & 17), or by taking units – for example a hexagon being one bar of music – and arranging them together into one composition (see Figures 16, 18, 19 & 20).

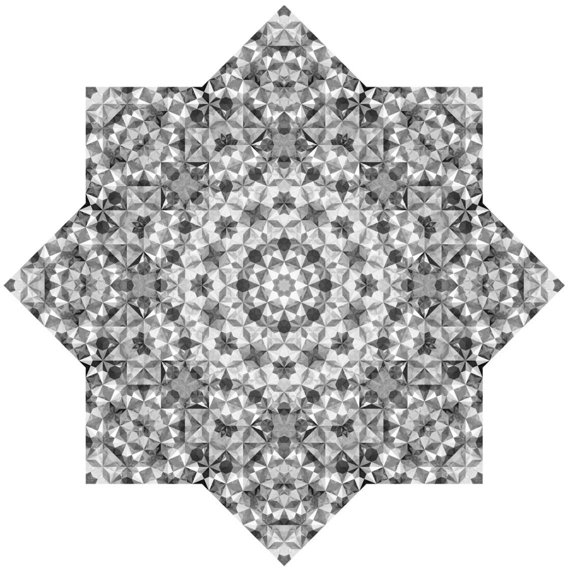


Figure 13: *Octagon I – Flute & Marimba*

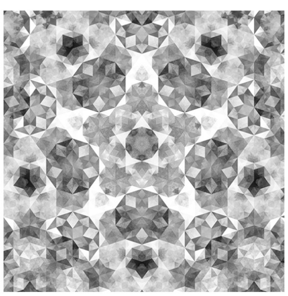


Figure 14: *Pentagon III 'Roundels' – Ensemble (detail)*

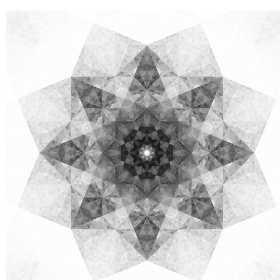


Figure 15: *D – Marimba*

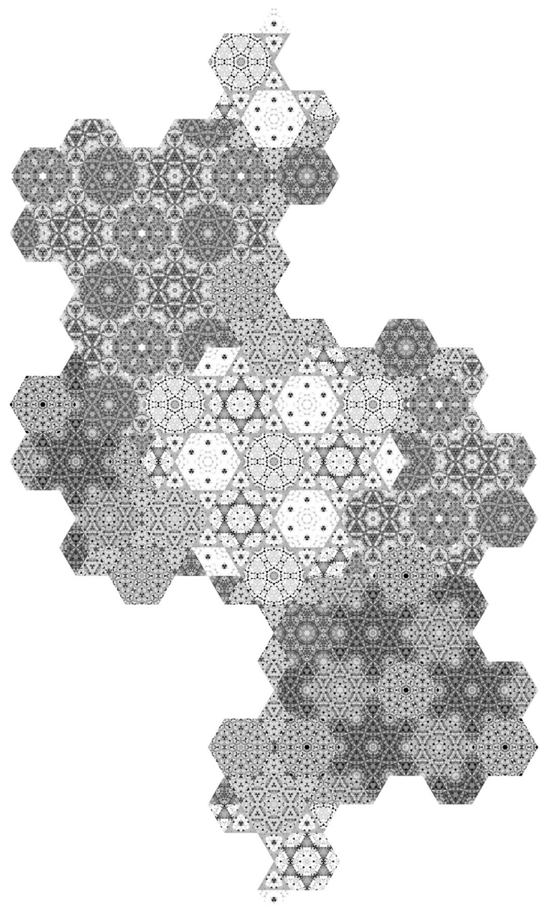


Figure 18: *Triangle Ensemble – Variation II*

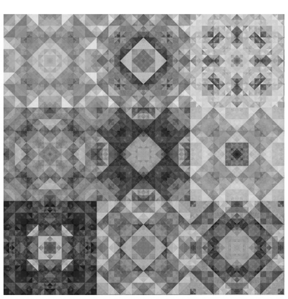


Figure 16: *Square – Ensemble (detail)*

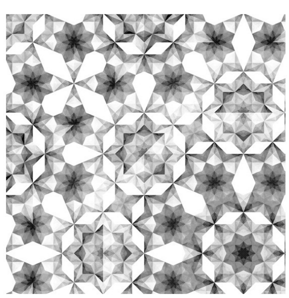


Figure 17: *Octagon III – Solo Conga – Study (detail)*

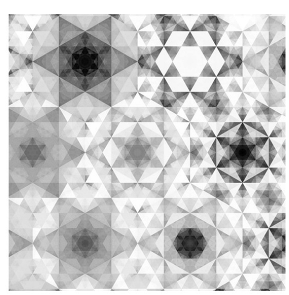


Figure 19: *Hexagon I – Ensemble (detail)*

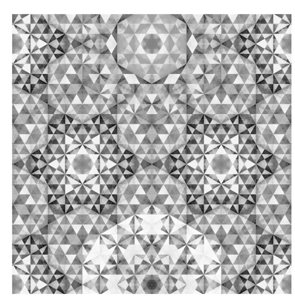


Figure 20: *Hexagon II – Cello & Percussion (detail)*

The above artworks are from the A Hidden Order collection by Sama Mara and Lee Westwood 2014

For further information about the A Hidden Order project including a description of the project, images of artworks and animations, please visit www.musicalforms.com.

The process for creating the pieces may either start as a musical idea then rendered and developed as an artwork, as in Figures 16, 18, 19 & 20. Or a begin as a visual artwork then transcribed into music to be further developed as a musical composition as in Figures 13, 14 & 17. After the composition / design process, the musical pieces were performed and recorded, and the subsequent audio files input into a computer program implementing the methods described here, creating the final animations and prints.

A satisfying outcome of this method is that when we further subdivide the unit of a beat /cell into smaller and smaller parts, the dynamics of the musicians performance and to some extent even the timbre of the instrument may be translated into form and texture, see Figure 15. The attack and decay of the sound of the marimba note can be seen in the fade out of the artwork from the darker centre. The subtle changes in hue (digital version only) are created by the complex pattern of overtones from the sound of the marimba.

Further Development

The grids so far discussed are created with a series of reflections, though grids constructed by rotations are equally of interest, one example being the Heighway dragon curve. There are a plentitude of other grids which may be explored for various time signatures and visual results. Beyond this, the realm of three dimensions without doubt demands attention: a possibility for this would be the 3-D Hilbert curve. As the time signature is linked to the symmetry of the grid, a free modulation to any time signature at will is incompatible with the demands of the grid's symmetry. To overcome this limitation with a continuous visual method would free up further musical possibilities. It would also be of interest to open up other families of pattern to a musical counterpart as there remains a great many which are not achievable using this grid method, one case in point being Islamic star patterns.

Conclusion

The search for an intrinsic relationship between visual art and music shall likely be a journey without end but with endless delights and beauties to be revealed, explored and enjoyed along the way. Here we have seen a relationship that reveals a one to one mapping between form and rhythm, and that allows for a great variety of expression in visual form and in musical language. This was achieved by working with the basic principles of harmony of sound and form – division of time and space. With the method outlined here it is possible to design artworks through musical composition as well as explore sonic themes through visual ideas. As the co-creator of ‘A Hidden Order’ Lee Westwood describes it: ‘...it should be remembered that, whether we are looking at the image, or we are hearing the musical composition, we are, in fact, perceiving the very same artistic “object”, from a different angle, or through the eyes of a different medium (sound or visual pattern)’ [9].

References

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[9] L. Westwood, & S. Mara, “A Hidden Order Suite” (no publisher, 2013). Score