# How to Draw Perspective Directly on a 3D Plane 

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#### Abstract

The aim of this Workshop is to learn hands-on to draw a building directly in perspective. You will experience how easy is to draw a view of The Arch of Defense (Paris) in accurate proportions, without the aid of blueprints. Unlike traditional methods, the entire process takes place on the perspective plane. All that it takes is the ability to see the building's points of interest in three-dimensions on this plane. We will begin with the basic notions of the Modular Perspective model and how the three modular scales of the perspective plane are read. The time lapse for each activity of the workshop is indicated (in minutes) at the beginning of each page.


## Introduction

The modular perspective model (see Figure 1) is composed of three planes, namely: plane of symmetry X (PLSX), plane of symmetry Y (PLSY), and the perspective plane (PPL). All three planes are read in modular coordinates from the observer ( OB ). In the model, it is self evident that human vision vanishes to a single vanishing point; whereas an object vanishes to as many vanishing points as parallel systems it may have. In practice, the modular coordinates $(\mathrm{X}, \mathrm{Y}, \& \mathrm{P})$ of any point given in space are read on the PPL by the scales: X -scale, Y -scale, and P -scale. In simple terms: X is the width, Y the height, and P the depth of a point.


Figure 1: The model of Modular Perspective.

## $\mathbf{0 0 - 1 5}$. Getting familiar with the PPL

In Figure 2 is shown the standard position of the X -scale, Y -scale, and P -scale. The former two scales are given in true modular dimension, while P -scale is given in modules that decrease from 0 to infinity. These scales are ready to be used graphically on the PPL75 (Perspective Plane, $75^{\circ}$ of Aperture of Visual Field). The math of my method, including the P -scale, is discussed in a previous work [1].

To start, participants will receive a printed sheet of the PPL75.
A module $(M)$ can be of any dimension you want it to be, either small or larger.
The X-scale (at the bottom margin) is used to read modular coordinates X, on the PLSX.
The Y-scale (at the right margin) is used to read modular coordinates Y , on the PLSY.
The P -scale (at the left margin) is used to place in depth both coordinates, X and Y (one at a time).
The $v p$ (at the center of the PPL) is the observer's vanishing point.


Figure 2: Square-12 (Sq12) is a square in perspective measuring $12 \times 12 \mathrm{M}$.

Drawing a square of $12 \times 12$ modules $(M)$ on the PPL 75, from here on abridged as Sq12.
The A $m$ VF (amplitude of visual field) is a secondary scale to measure either X or Y values bigger than $\pm 5.00 \mathrm{M}$. It's placed within the PPL in both positions vertical and horizontal (see Fig. 2). First. Measure on the $\mathrm{A} m \mathrm{VF}$ scale, $\mathrm{X}=+6.00 M$ (right), and carry it to the $v p$. Measure on the $\mathrm{A} m \mathrm{VF}$ scale, $\mathrm{X}=-6.00 \mathrm{M}$ (left), and carry it to the $v p$. This would be the width of Sq12 $=|12.00 \mathrm{M}|$.
Second. Trace a horizontal line from $\mathrm{P}=4.00 \mathrm{M}$ across the PPL , and another horizontal line from $\mathrm{P}=$ 16.00 M across the PPL. Notice that the subtraction of both P values $=|12.00 \mathrm{M}|$.

Third. Give shape to Sq12 (shaded in grey).

15-25. To lay out the Defense plan in foreshortened view mark the sequence ( $9 M, 3 M$ ) at all sides of Sq 12 . To measure, use either the $\mathrm{A} m \mathrm{VF}$ or the X -scale.
First. Measure on the frontal side of Sq12, 9.00 M , from right to left, and draw a line to the $v p$ until it crosses the frontal side (leave a stroke).
Second. Measure on the left side of Sq12, 9.00 M. Use for this the P-scale at $\mathrm{P}=13.00 \mathrm{M}$.
Third. Measure on the rear side of $\mathrm{Sq} 12,9.00 \mathrm{M}$, from left to right, and draw a line to the $v p$ until it crosses the rear side (leave a stroke).
Fourth. Measure on the right side of Sq12, 9.00 M . Use for this the P -scale at $\mathrm{P}=7.00 \mathrm{M}$.
Fifth. Inscribe a new square from the marked points. This would be the Defense ideal plan in foreshortened view, from here on as SqD.


Figure 3: The Arch of Defense, square plan (SqD).
As we know, The Arch of Defense is about 110 meters by side and so of height.
By the Pythagorean theorem, the side length of SqD result $\approx 9.5 \mathrm{M}$. Notice that all inner angles of SqD are of $90^{\circ}$. On the PPL, the $9.5 M \times 9.5 M$ SqD represents the actual plan of The Arch of Defense that in reality measures $110 \mathrm{~m} \times 110 \mathrm{~m}$ (meters); which means that $1 M=11.6 \mathrm{~m}$.

Now, again in modules ( $M$ ), the proportions of the building that you should know in advance are:
a) The outer glazing-walls of the building measures 9.5 M .
b) The width of the Arch's legs is 1.7 M on the X -scale, and on the oblique side of $\mathrm{SqD} \approx 1.8 \mathrm{M}$. *
c) The interior plaza of the Arch rises 1.0 M from the esplanade (Le Parvis).
d) The thickness of the roof of the Arch is 1.3 M .

* (As the ratio $9.5 / 9.0=1.05$, then we have: $1.05 \times 1.7 \approx 1.8 M$ ).

25-40. Now, to set the width of the Arch's legs of 1.8 M on the foreshortened side of SqD, do as follow:
First. Measure 1.70 M on the X -scale, from the left corner to the right, and also from the right corner to the left (leave strokes).
Second. Carry visual rays from the strokes to the $v p$; and again leave marks where these visual rays intersects the foreshortened frontal side of SqD.
Third. From the side of SqD, carry the width of the legs to the $a v p$. (The avp lies on the $v h$, where the lateral sides of SqD intersect).
Fourth. Trace both diagonals of SqD, of which, one of them runs at the divp.
Fifth. Then, where these diagonals cross the inner lines of the arch legs, a second SqD is formed. This would be base of the inner cube of Defense.


Figure 4: Layout of the Defense legs.

## Remarks

$v p$, vanishing point (is always at $\mathrm{Y}=0.00 M$ on the $v h$ ).
$v h$, visual horizon (is always at your sight level).
avp, asymmetrical vanishing point (which in this case lies on the $v h$ ).
divp, diagonal vanishing point (it's used to construct the $45^{\circ}$ angle of the architectural plan).

40-55. Next, to shape the volume, do the following carefully.
First. Draw vertical lines from the corner points of both squares.
Second. Measure on the Y -scale, from the $v h, \mathrm{Y}=-3.70 \mathrm{M}$ (downwards), and $\mathrm{Y}=+5.80 \mathrm{M}$ (upwards), that is $|9.5 \mathrm{M}|$. These would be the outer edges' height.
Third. Measure on the Y -scale, from the $v h, \mathrm{Y}=-2.70 \mathrm{M}$ (downwards), and $\mathrm{Y}=+4.50 M$ (upwards). These would be the inner edges' height.
Fourth. Look for the diagonal. Now, trace a horizontal line from each corner point (of both squares) until it crosses the diagonal.
Fifth. Next, from the diagonal, raise 8 new vertical lines (corner points of the volume).


Figure 5: The corner points delimit the shading areas.

## Remark

Keep in mind that you have drawn two sets of vertical lines, one belonging to the figure points, and the other along the diagonal (see Glossary). The function of the diagonal is to carry P from any pair $(\mathrm{X}, \mathrm{P})$ to its corresponding Y . In other words, the two pairs of coordinates $(\mathrm{X}, \mathrm{P})$ and $(\mathrm{Y}, \mathrm{P})$ that a point has, must have the very same P . This is the key to establish the heights of all corner points.

55-85. Before shaping the volume, keep in mind that the height of the observer is $|3.7 M|$.
The square plan of both cubes is given by their corner points. As we know, there is an X and a P for each corner point but to simplify the process, here we will use only its P value.
First. Select a vertical of a corner point in plan (let's say the nearest one to the observer). Then, follow the $\mathrm{P}=4.00 \mathrm{M}$ horizontal line up to the diagonal; from there, turn up and follow its vertical line until intersecting with both $\mathrm{Y}=-3.7 \mathrm{M}$ and $\mathrm{Y}=+5.8 \mathrm{M}$, leave strokes.
Second. Now, from both Y's marks draw horizontal lines until intersecting with the vertical of the corner point, to thus determine the edge of the outer cube we are looking for.
Third. Take a corner-point at a time and repeat this very same procedure over and over until get the volume shaped. It would be easier if you first draw the outer cube and then the inner cube. Make sure of proceed systematically in both cases.


Figure 6: The Arch of Defense drawn in accurate proportions.

## Remark

The observer distance to the building is deduced as follow: $\mathrm{P}=4 M+d(6.51 M)=10.51 M=121.9 \mathrm{~m}$. Where $(d)$ is the distance from the observer to the PPL, which for an aperture of visual field of $75^{\circ}$ is of 6.51 M . Other interesting measures are the following. The nearest corner of the building is at 121.9 m from the observer. The $v h$ is at 42.9 m high ( $3.7 M \times 11.6 \mathrm{~m}$ ). And the viewing foreshortened angle is of $18.43^{\circ}\left(\tan ^{-1}, 3 / 9\right)$.

85-90. The foreshortened angle in this view (Figure 8), of $19.5^{\circ}$, was set for the purpose to occlude the left lateral surface [2]. This is why I decided to change it a little bit to make more comprehensible its perspective construction, for the practice of this workshop.


Figure 8: La Grande Arche (Paris), Johan Von Spreckelsen.

## References

[1] T. García-Salgado, A Modular Perspective Model Vs. Vectorial Models, Leonardo, Vol. 21, No. 3, pp. 277-284, 1988. (See equation of P-scale in Note 9, p. 283).
[2] T. García-Salgado, Modular Perspective for Architects \& other Related Professionals (Mexico: author's pilot edition, 2014, ISBN 978-607-00-8057-9), p 96.

## Glossary

Diagonal, it is any of the "diagonal lines" of the PPL. These diagonals are actually visual rays running parallel to the observer sightline.
distance $(d)$. It is the interval between the observer station $(\mathrm{OB})$ and the PPL. It therefore, the aperture of visual field depends on (d). Notice that the term "distance" is often used to refer the distance between the observer and an object, which is not the same concept of "distance" here defined.

Perspective Plane (PPL). It is the plane onto which objects as perceived by the observer are projected in perspective. The PPL construction is made by means of the modular scales: X-scale, Yscale, and P-scale.

Plane of Symmetry X (PLSX). It is the plane onto which modular coordinates ( $\mathrm{X}, \mathrm{P}$ ) are measured.
Plane of Symmetry Y (PLSY). It is the plane onto which modular coordinates ( $\mathrm{Y}, \mathrm{P}$ ) are measured. Vanishing Point ( $v p$ ). It is a point at infinity where the observer's visual rays meet, though these rays actually run parallel.

